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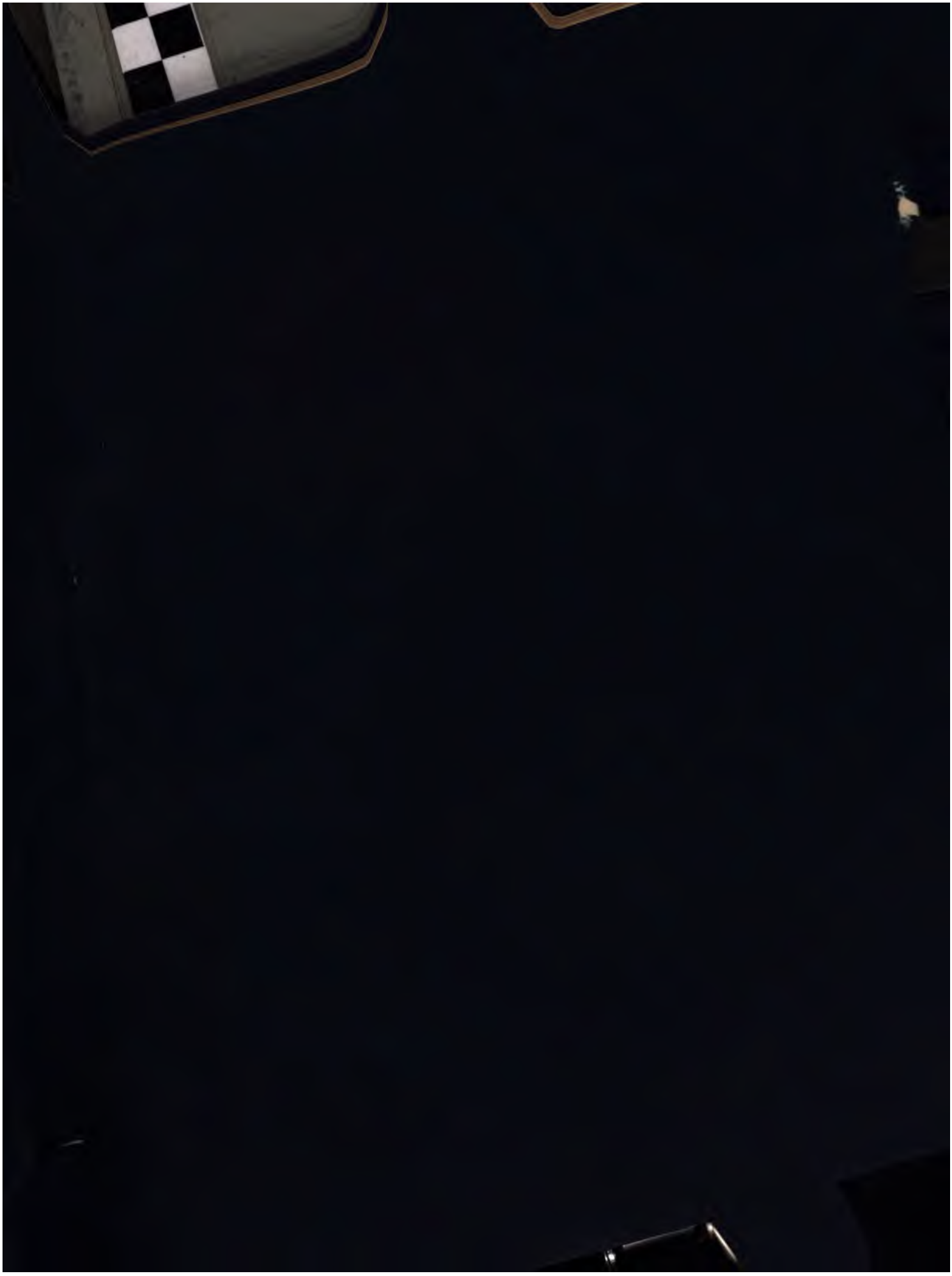

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PREFACE.

THIS work has been compiled to supply a want long experienced by science teachers and their pupils, and is published at a price within the means of the humblest artisan. By uniting Plane and Solid Geometry with Orthographic, Isometrical, Parallel, and Linear Perspective Projections, in a clear and condensed form, it will prove of great assistance to workmen in their several branches of trade, especially tin-plate and metal workers, pattern makers, fitters, smiths, engineers, carpenters, joiners, masons, and others, surveyors and draughtsmen. With the exception of the terms explained in the definitions, technical and compound words have as far as possible been avoided, in order that the subject may be easily and thoroughly comprehended by persons having no previous knowledge of Geometry.

Although there are various methods of working out these problems, the simplest and best that will apply in all cases has been adopted.

The work is *descriptive* and *not demonstrative;* long experience having proved how few students in our schools and classes have either the preliminary training, time, or application to enter into the theory: they learn rather *how* to do it, than *why* it is executed in that particular form.

The best examples in each division have been selected, so as neither to limit nor encumber the subject. It is hoped that the description to each, although brief, will be found complete in itself; but in some cases, where incompleteness may be apparent, a reference to other figures mentioned will suffice. A copious description would often tend to make it more confusing. As the great value of Geometry lies in its accuracy, it cannot be too urgently impressed upon the student, that figures should be drawn as large as possible, with neatness and care; for, without these essentials, errors will constantly occur. It is suggested that, where doubt is entertained of the truth of any solution worked out by the student, exaggerated examples should be taken, which will multiply the error, if such exist, and serve as an exercise to impress the method more fully on the mind of the pupil.

The work is divided into sections, as follows:—

PRACTICAL PLANE GEOMETRY.

Definitions. Elementary Constructions of Plane Figures. Division and Transformation of Plane Figures.

PRACTICAL SOLID GEOMETRY.

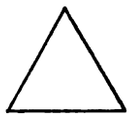
Orthographic projection of Lines, Planes, and Solids. Penetrations and Intersections of Solids. Conic and other Sections. Envelopes of Solids, with the development of Lines and Points upon their surfaces. Projection of Shadows and Cast Shadows. Isometrical Perspective Projection. Parallel Perspective Projection. Linear Perspective Projection.

Definitions.

Plate 1.

Fig. 1

Fig. 2



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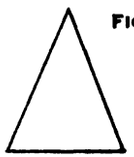

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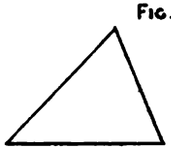

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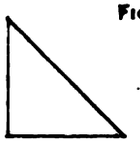

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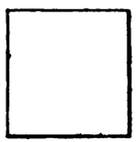

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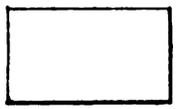

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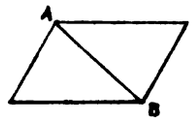

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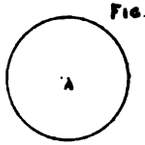

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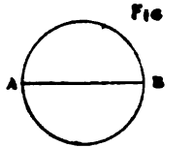

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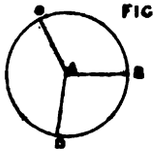

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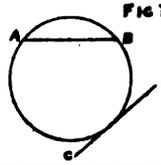

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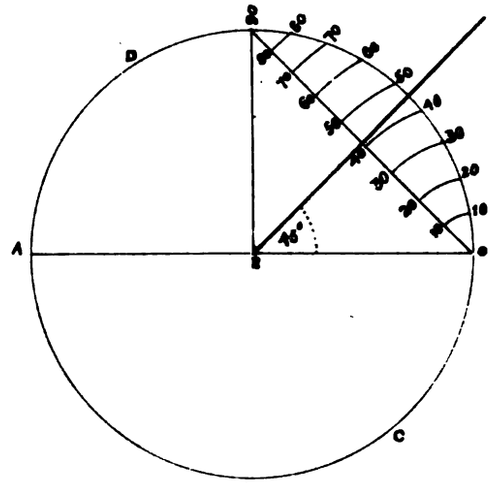

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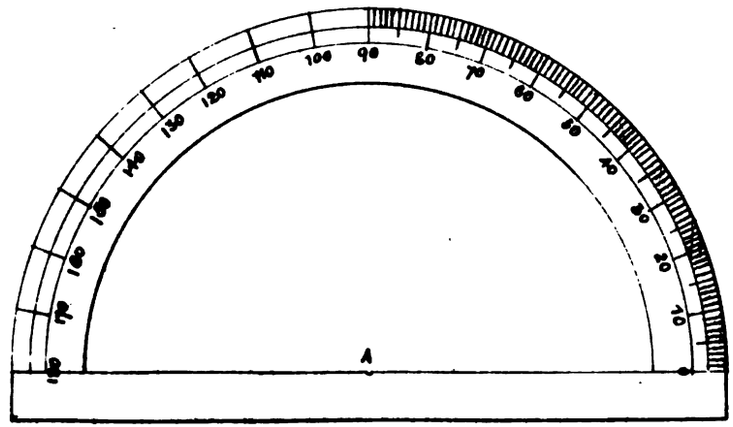

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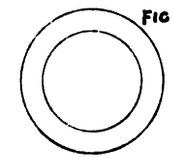

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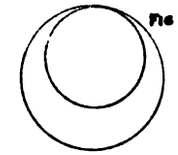

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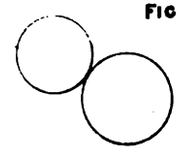

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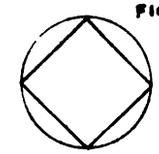

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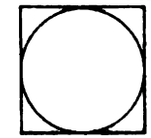

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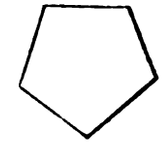

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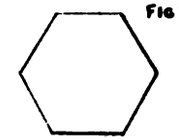

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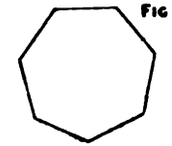

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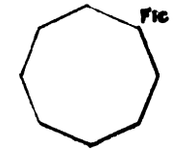

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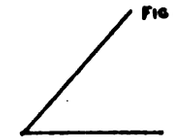

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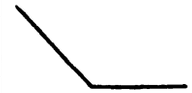

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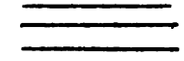


WILLCOCK and HUDSON's Geometry.

PRACTICAL PLANE GEOMETRY.

DEFINITIONS.

PLATE I.

FIG. 1.

A *right angle* is when one line is perpendicular to another, or contains an angle of 90 degrees.

FIG. 2.

An *equilateral triangle* has three sides equal.

FIG. 3.

An *isosceles triangle* has two sides equal.

FIG. 4.

A *scalene triangle* has all its sides unequal.

FIG. 5.

A *right angle triangle* contains a right angle.

FIG. 6.

A *square* is a four-sided figure, having all its sides equal, and its angles right angles.

FIG. 7.

A *rectangle* is a four-sided figure, having all its angles right angles, and its opposite sides only equal.

FIG. 8.

A *parallelogram* is a four-sided figure, having its opposite sides parallel. A B is called the diagonal.

FIG. 9.

A *circle* is a plane figure bounded by one continuous curve line, called the circumference, and is such that the circumference is at all points equally distant from the centre A within it.

FIG. 10.

A line A B drawn through the centre of a circle and terminating on each side at the circumference, is a *diameter.*

FIG. 11.

The line A B is a *radius* of the circle B C D—A C and A D are also radii. The space enclosed by two *radii*, and a portion of the circle is termed a *sector.*

FIG. 12.

An *arc* of a circle is any part of the circumference, as A B. A *chord* of a circle is a straight line joining the extremities of an arc, as A B. A *segment* of a circle is a space enclosed by an arc and chord, as A B. A *tangent* to a circle is a straight line which touches it but does not cut it, as C D.

FIG. 13.

Circles are *concentric* when they have the same centre.

FIG. 14.

Circles are *eccentric* when they have not the same centre.

FIG. 15.

Circles are *tangential* to each other when they touch at their circumference, but do not cut.

FIG. 16.

A square, or any other figure, with a greater or less number of sides is *inscribed* in a circle when all its angles touch the circumference but do not cut.

FIG. 17.

A circle is *inscribed* in a straight sided figure when it touches all its sides.

FIG. 18.

A *pentagon*, or five sided figure.

FIG. 19.

A *hexagon*, or six sided figure.

FIG. 20.

A *heptagon*, or seven sided figure.

FIG. 21.

An *octagon*, or eight sided figure.

FIG. 22.

An *acute angle*, less than a right angle.

FIG. 23.

An *obtuse angle*, greater than a right angle.

FIG. 24.

Parallel lines if produced will never meet.

FIG. 25.

To construct a scale of chords.

Draw any circle A C D, and find the diameter A B O. Erect a perpendicular at B, and divide one quarter of the circumference from 0 to 90 into nine equal parts. Then each of these divisions represents ten degrees; they should each be divided into ten equal parts. The circumference of a circle contains 360°, therefore the quadrant or quarter circumference will contain 90°. Draw the chord line 0, 90; and from 0 as a centre, with radii 0, 10; 0, 20; 0, 30, etc., describe arcs to cut the chord, and figure these points 10, 20, 30, etc. Then this line is called the scale of chords, by means of which any angle may be set off. Thus, for an angle of 45° lay down a line,

and, from a point in it as a centre, with a radius always equal to 0, 60, on the chord draw an arc. From the point where this arc cuts the line as a centre, and with a radius equal to 0, 45, on the chord cut the arc. Through this point (where the arcs cut) draw a line from the end of the base which will form the required angle.

See its application in Figure 5, Plate 2.

FIG. 26.

To construct a Sector.

A Sector is an instrument made of metal, horn, or paper, and usually of the form shown[1], the degrees of the semicircle being accurately divided and marked upon its edge. It is used for setting off and measuring angles instead of the scale of chords.

[1] Sectors are also made rectangular in form.

RULES AND AXIOMS.

All parallelograms, whether right or oblique-angled, having equal bases and equal altitudes, have equal areas. See Fig. 73.

All parallelograms having equal altitudes, have to each other the same ratios (of area) as their bases. See Fig. 73.

All parallelograms having equal bases, have to each other the same ratios as their altitudes. See Fig. 73.

All triangles are equal to parallelograms of equal altitudes and half the base; or of equal base and half the altitude. See Fig. 75.

All triangles having equal bases and equal altitudes, have equal areas. See Fig. 74.

The area of a circle is equal to a parallelogram, two of the sides of which are equal to half the circumference, and the other two to the radius of the circle; or the whole circumference and half the radius. See Figs. 82 and 86.

All the interior angles of any triangle are together equal to two right angles. See Fig. 28.

All the exterior angles of any triangle are together equal to four right angles. See Fig. 29.

PLATE II.

FIG. 1.

To bisect or divide into two equal parts a given straight line.

With any radius greater than half the given line A B, describe arcs of circles from the extremities A and B, cutting each other at the points C and D. Draw a line through C and D. This line will cut the given line into two equal parts and will be perpendicular also.

FIG. 2.

To draw a perpendicular to a straight line from a point without it, as C.

From the point C, with a sufficient radius, cut the given line A B, and from these points describe arcs cutting at D. Draw the perpendicular C D.

FIG. 3.

To draw a perpendicular to a straight line, from a given point in any part of that line.

From any centre C above the given line, describe a circle passing through the given point B, and cutting the given line in A. Draw A C, and produce it to cut the circle at D. Draw B D, the perpendicular required.

FIG. 4.

To construct a rectilinear angle equal to any given one, or to copy a given angle.

Let C A B be the given angle. Draw a line A′ B′. With any radius A C, draw an arc C B, and with the same radius describe C′ B′. Take the chord of the arc C B, and transfer it to B′ C′. Draw A′ C′.

FIG. 5.

To make an angle of a given number of degrees.

Upon a line A B, describe an arc B C, with a radius always equal to 60° on the scale of chords or sector (Figure 25, Plate 1). Take the length of the number of degrees on the scale of chords or the circle, and mark it from B to C. Draw A C, which will form the required angle with A B.

FIG. 6.

To bisect an angle.

Let C A B be the angle. On A, as a centre, with any radius, cut the sides at B and C, and on B and C, as centres, describe arcs cutting at D. Draw A D, dividing the angle into two equal parts.

FIG. 7.

To bisect the inclination of two lines, A B and C D.

Draw A C and B D parallel, bisect the part between the two given lines, and draw E F through the points of intersection.

FIG. 8.

To draw a line through a point between two inclined lines, that, if produced, would pass through the point of intersection of the two lines.

Let A B and C D be the given lines, and E the point. From E draw perpendiculars E F and E G to the given line (as per Fig. 2). Bisect the angle G E F (see Fig. 6). Draw line H E, which would pass through the point of intersection of the two lines produced.

FIG. 9.

To draw a line through a given point parallel to a given line.

From C, the given point, describe the arc A D, cutting the given line at A B. From A, with the same radius, describe the arc C B. Set off A D equal to C B. Draw the parallel through the point C D.

FIG. 10.

To construct an equilateral triangle, the length of the base being given.

On the ends A B of the given base, with A B as radius, describe arcs cutting at C, and draw A C, B C; then A B C is the triangle required.

FIG. 11.

To construct an isosceles triangle, the length of the base and one side being given.

On the ends of the base A′ B′, equal to A B, with

Plate 2.

FIG. 1

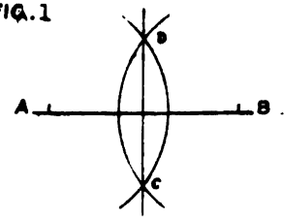


FIG. 2

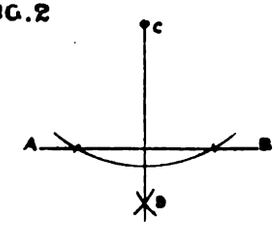


FIG. 3

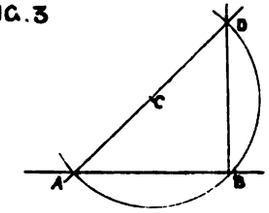


FIG. 4

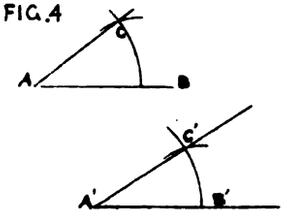


FIG. 5

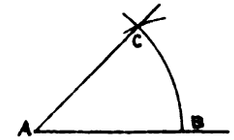
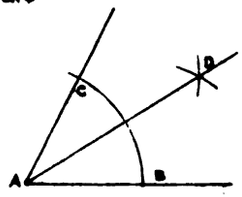


FIG. 6



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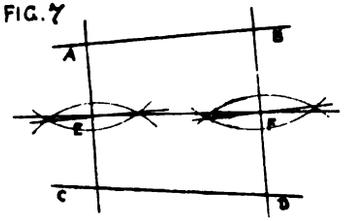

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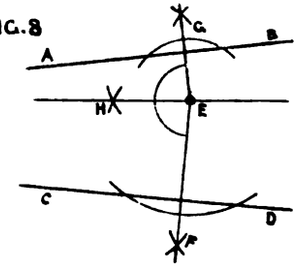


FIG. 9

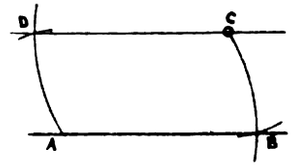


FIG. 10

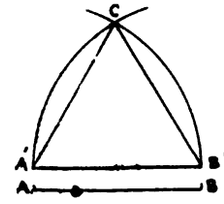


FIG. 11

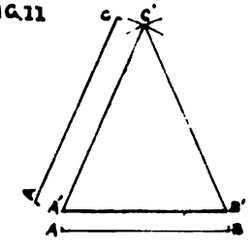


FIG. 12

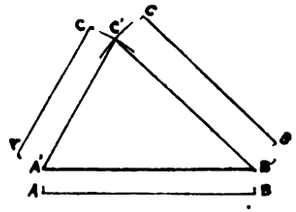


FIG. 13

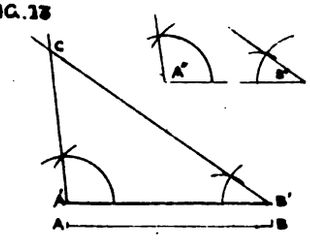


FIG. 14

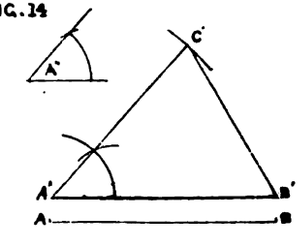


FIG. 15

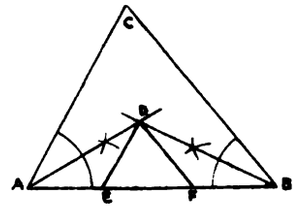


FIG. 16

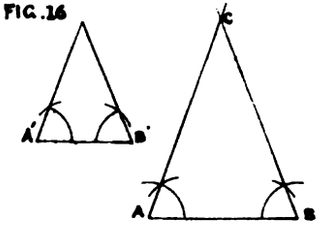
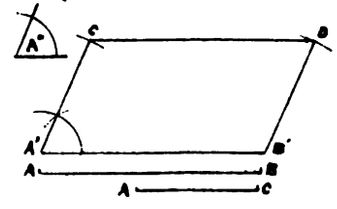


FIG. 17



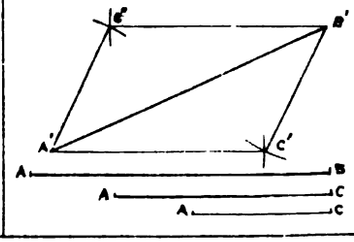


FIG. 19

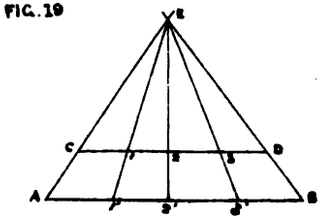
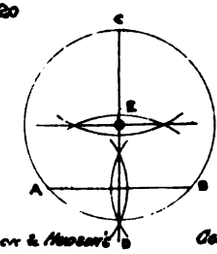


FIG. 20



Wellcome & Mundell Geometry.

the given side A C as radius, describe arcs cutting at C'. Draw A' C', C' B'; then A' B' C' is the triangle required.

FIG. 12.

To construct any triangle, the length of the three sides being given.

At the end A of one side, A' B' equal to A B, with the side A C as radius, describe an arc, and upon B', with the side B C as radius, cut the arc at C'. Draw A' C', C' B', then A' C' B' is the triangle required.

FIG. 13.

To draw any triangle when the base and its two adjacent angles are given.

Let A'' and B'' be the angles, and A B the given base. Draw A' B' equal to A B. At A' make an angle equal to A'', and at B' make an angle equal to B''. Produce the lines till they meet at C.

FIG. 14.

To draw any triangle when two sides and the contained angle are given.

Let A'' be the angle, and A B, A C the two sides. Draw A' B' equal to A B at one end of this base, as A'. Construct an angle equal to A''. Produce the other side, making it equal to A' C'. Join B' C', which will complete the triangle required.

FIG. 15.

To draw a triangle similar to another given one, and having a perimeter equal to a given line.

Let A B C be the given triangle, and A B, equal to the given perimeter, bisect two angles A and B by lines cutting at D. Draw D E and D F parallel to A C and B C. Then the triangle D E F is made similar to the given one, and has its perimeter equal to A B.

NOTE.—The perimeter is the total length of the sides.

FIG. 16.

To draw an isosceles triangle when the base and adjacent angles are given.

Let A' B' be the given base and angles. Draw A B equal to the given base, and at A erect an angle equal to A', and another at B, making the angles at A' and B' both alike; produce the lines to cut at C.

FIG. 17.

To draw a parallelogram, the sides and one angle being given.

Let A B and A C be the length of two adjacent sides, and A'' the given angle contained by them. Draw a straight line, set off A' B' equal to A B; from A' draw A' C equal to A C', and forming an angle with A' B' equal to A''. From B', with A' C' as radius, describe an arc; and from C, with A B as radius, cut this arc at D. Join C D and B' D to complete the parallelogram.

FIG. 18.

To draw a parallelogram, the sides and its diagonal being given.

Let A B be the given diagonal, and A C and A D the sides. Draw A' B' equal to A B, and from A' and B', with A C and A D as radii, describe arcs intersecting at C' and C''. Join A' C', C' B', B' C'', and C'' A', and the parallelogram will be completed.

FIG. 19.

To divide a line in the same ratio as a given one.

Let A B be the given line, and C D the ratio, as at 1 2 3. Join the ends of these lines, producing A C and B D, till they cut at E. Draw lines from E through 1 2 3, and cutting A B in the required ratio. The lines A B and C D must be drawn parallel, at any convenient distance apart.

FIG. 20.

To find the centre of a circle.

Draw any chord as A B, bisect it by the perpendicular C D, bounded both ways by the circumference, and bisect C D for the centre E.

FIG. 21.

To erect a perpendicular to a curve or arc.

Let A B be a given curve, and C the point where the perpendicular is to be drawn. Mark off two equal distances from C to A, and C to B. From C and B with any radius draw arcs, cutting at D. Draw C D, the line required.

FIG. 22.

To draw a circle passing through three given points.

Join the given points A B C, and bisect A B and B C by perpendiculars cutting at D. On D as centre describe a circle passing through the three given points.

FIG. 23.

To draw a tangent to a circle at a given point in the circumference.

Through the given point A draw the radial line C A D, and the perpendicular F E, for the tangent required.

FIG. 24.

To draw a tangent to a circle from a given point not in the circumference.

From C, the given point, draw a line to the centre of the circle D. Upon this line draw a semicircle D A C. From C draw a line through A, where the circles cut, forming a tangent with the given circle A B D.

FIG. 25.

On a given line to draw the arc of a circle, to contain a given angle.

Let A B be the given line and C the angle. At each end of the given line A B, and on the opposite side of the line to that on which the arc is to lie, construct an angle equal to the given one C. Draw A D and B D perpendicular to A C and B C, then the point D, where these lines cut, is the centre of the required circle, and D A is the radius. When the given angle contained by the arc is less than a right angle, the centre D falls above the line; but if the given angle be greater than a right angle, it will fall on the contrary side of the line.

NOTE.—An arc of a circle is said to contain a given angle, when lines drawn from each end of the chord A B to any point in the circumference contain that angle as at E. The angle at E, formed by the lines A E, B E, is the same, to whatever point in the circumference of the arc these lines are drawn.

FIG. 26.

To inscribe a circle in any triangle.

Bisect two of the angles, as A B, of the triangle A B C, by lines cutting at D. Produce one of these

lines, as B D to E. On D as centre, with D E as radius, describe the circle required.

FIG. 27.

To draw a circle about any triangle.

Bisect any two sides as A C and C B by perpendiculars cutting at D. On D, with D A as radius, describe the circle A B C required.

FIG. 28.

To inscribe any triangle in a circle similar to a given triangle.

Let A B C be the given circle, and D E the triangle. Draw a tangent to the given circle, as D′ E′. At the point of contact C make an angle B C D′ equal to angle D, and an angle A C E equal to angle E. Produce the sides of the angles to cut the circle at A and B. Join A B. Then A B C will be the triangle required, having its angles equal to those of the given one.

FIG. 29.

To circumscribe any triangle about a circle, similar to a given triangle.

Let B C D be the given circle, and A′ A″ the triangle. Produce the base of the given triangle A′ A″, so as to determine two external angles. Draw any radius, as A B, of the circle, and another A C, making angle B A C equal to one of the external angles A′ of the given triangle. Draw another radius A D, making angle C A D equal to the other external angle A″ of the triangle. Draw tangents or perpendiculars to the radii, at the points B C D, cutting at E F G, and forming the required triangle.

FIG. 30.

To draw a circle to touch two given circles, and one of them in a given point.

Let D and E be the centres of the given circles, and A the given point in one of them. Draw from A, the proposed point of contact, a radius A D. In the second circle draw a radius E C parallel to A D. Join A C, producing it to a point opposite to A, as B. Draw lines A D, B E, producing them until they cut in F. On F as centre, with F A as radius, describe the circle required.

FIG. 31.

To draw a circle to touch two given ones and to include one, also to touch one in a given point.

Let B and C be the centres of the given circles, and A the given point in one of them. Draw B A and prolong it to D, making A D equal to the radius of the other circle C. Join C D, and bisect it by a perpendicular cutting A B in E. Then E A is the radius of the required circle.

FIG. 32.

To draw a circle to touch two given ones and to exclude both, also to touch one in a given point.

Let B and C be the centres of the given circles, and A the point where the required circle is to touch one of them. Join the centres B and C by a line. Bisect it by a perpendicular and produce it on the side where the circle is to be drawn. Join B, the centre of the circle, with A, the given point, and produce this line to cut the perpendicular at E. Then E will be the centre of the required circle, and E A the radius.

FIG. 33.

To draw a circle of a given radius to touch two given circles and exclude both.

Let B and C be the given circles. From B as a centre with a radius equal to B A, added to the radius of the circle to be drawn (A E), draw an arc. Then from C as a centre with a radius equal to C D, added to the radius of the circle to be drawn, cut the other arc in E. Draw the circle E A D as required.

FIG. 34.

To inscribe a square in any given triangle A B C.

From the vertical angle C draw a line C D, perpendicular to the base A B; this is the altitude or vertical height of the triangle. From C draw a line C E, parallel to the base A B, and equal to the altitude C D. Draw from the angle A, opposite E, a line E A, cutting the side of the triangle C B in F. From F draw F G, perpendicular to the base A B. Draw line F H parallel to A B, and H I parallel to F C, forming the square required.

NOTE.—The perpendicular C D may be drawn from either angle to the opposite side, provided that neither of the angles at the extremity of that side, or base, are greater than a right angle.

FIG. 35.

To inscribe a rectangle in any triangle, such rectangle having a base not greater than the longest side of the triangle.

Let A B C be the given triangle, and A′ D′ one side of the rectangle. Set off A D on the base of the triangle equal to A′ D′. From D draw a line parallel to one side, as D F. From the point F, where the line D F cuts the side, draw line E F parallel to the base. From E and F drop perpendiculars to the base, which completes the rectangle.

FIG. 36.

To inscribe a rectangle in any circle that shall represent the strongest form of beam which can be cut out of a tree.

Let A B be the diameter of any given circle. Divide A B into three equal parts. At the points 1 and 2 erect perpendiculars on opposite sides of the line (A B) to cut the circle, as at C and D. Join A C, C B, B D, and D A. Then the rectangle A C B D will be the best beam that can be cut out of the tree represented by the circle.

FIG. 37.

To construct an isosceles triangle on a given base and with a given vertical angle.

Let A B be the given base, and C the vertical angle. Produce the base A B on one side. Make an angle at C′ equal to the given angle (C). Bisect the remainder of the angle C′ E D by a line B H. Draw line A H, making the same angle with the base as B H, and cutting at H. That is, make angle F A G equal to D B E. Then the angle H will be equal to C.

FIG. 38.

To construct any polygon upon a given side as A B.

Produce the given side A B to C, making B C equal to A B. From B as centre with B A as radius draw a semicircle A D C. Divide the circumference of the semicircle into the same number of parts as the polygon is to have sides, in this case five. From B

FIG. 21

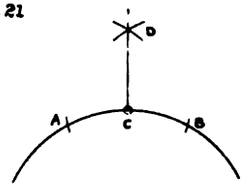


FIG. 22

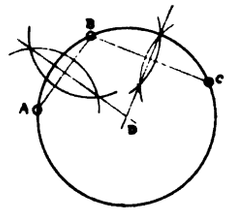


FIG. 23

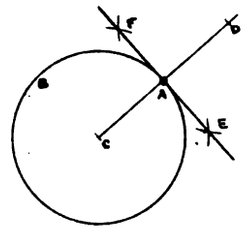


FIG. 24

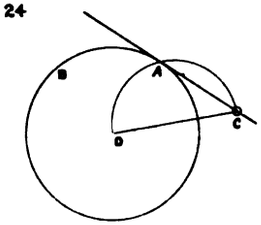


FIG. 25

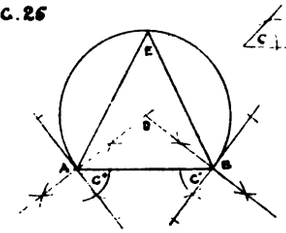


FIG. 26

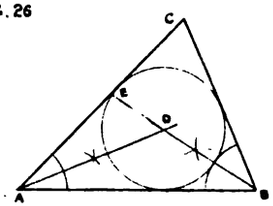


FIG. 27

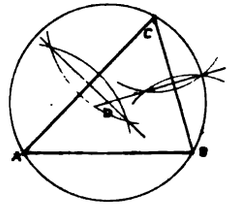


FIG. 28

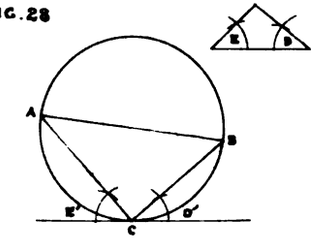


FIG. 29

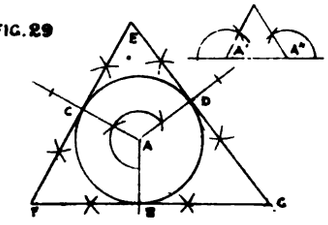


FIG. 30

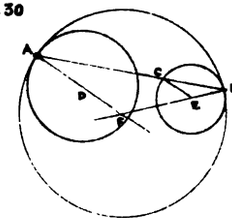


FIG. 31

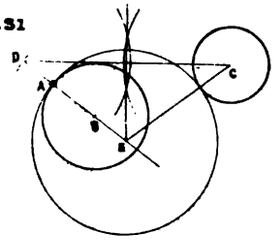


FIG. 32

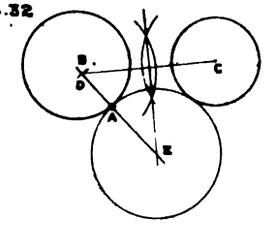


FIG. 33

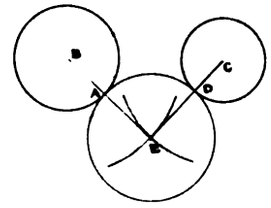


FIG. 34

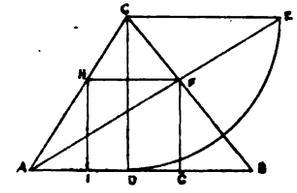


FIG. 35

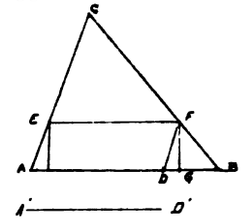


FIG. 36

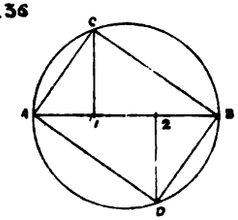


FIG. 37

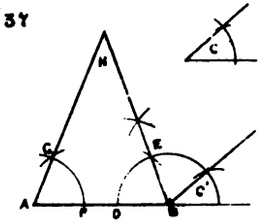


FIG. 38

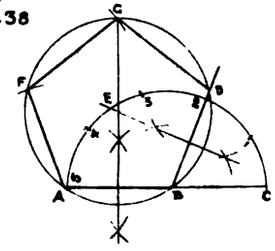


FIG. 39

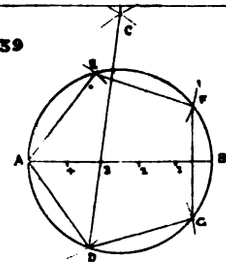
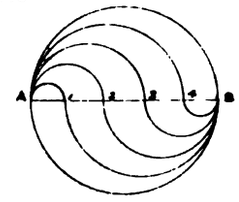


FIG. 40



draw a line through the second division, as B D. Bisect A B and B D by perpendiculars, cutting at E, the centre of the polygon. Draw the circumscribing circle from E as a centre, and E A as radius. Take the length of one side and step it round the circumference. Join these points, completing the polygon. The position of the other sides may also be determined by drawing lines from B through 3, 4, etc., cutting the circle in F G, etc.—then joining these points.

FIG. 39.

To inscribe any polygon in a given circle.

Draw the diameter A B, and divide it into the same number of parts as there are to be sides in the polygon. From A and B as centres, with the diameter as radius, draw two arcs, cutting at C. Draw a line from C through the second division (in all cases) from either end of the diameter, as at 3, and produce it to cut the circle at D. Then A D is the length of one side of the polygon. Take the length of this side A D, and step it round the circle, as G F E. Complete the polygon by joining these points A D G F E.

FIG. 40.

To divide a circle into any number of parts equal in area and perimeter.

Divide the diameter of the given circle A B into the number of parts required, as 5. Draw a semicircle on A 1, with half this line as radius; and a semicircle on the remainder of the diameter 1 B, but on the opposite side, with half 1 B as radius. Draw a semicircle on A 2, A 3, and A 4, on the same side of the diameter. Draw semicircles on the opposite side of the diameter, as 2 B, 3 B, and 4 B. Then the spaces enclosed by these curved lines are equal in area, and the total length of each curve line will be alike.

FIG. 41.

To draw a rectangle similar to a given one, but having a different height, or base.

Let A B C D be the given rectangle and A E the base of the required triangle. Draw the diagonal A D and a line from E, parallel to B D, to cut the diagonal at C. Draw a line from G to F, parallel to the other sides. Then A E G F is the rectangle required. If the side A F is given, draw a line from F, parallel to C D, to cut the diagonal at G. Draw G E parallel to D B. Any rectangle drawn inside or outside the given one with two sides cutting the diagonal, and the other two parallel with the sides of the given rectangle, must be similar; their sides being in the same ratio.

FIG. 42.

To draw a figure similar to a given one, but of a different size.

Let A B C D E F be the given figure, and A K the base of the required figure. Then draw radial lines from A to all the angles, as A C, A D, A E, and A F. Draw a line from K, parallel to B C, to cut the first radial at J. From J draw the line J I, parallel to C D. From I draw a line I H, parallel to D E. Draw line H G, parallel to E F. Then the figure A G H I J K will be similar to the given figure, and will have its sides in the same ratio; therefore its angles are the same. If the side A K be greater than A B (the given figure), produce the radials beyond the figure, and proceed as before described.

FIG. 43.

To divide a circle into any number of concentric parts.

Let A C be the diameter of the given circle. Divide B C, the radius, into the required number of parts, as 5. Draw a semicircle on B C and erect perpendiculars at the points 1 2 3 4, cutting the semicircle in 1′ 2′ 3′ 4′. Then from the centre B, with B 1′, B 2′, B 3′, and B 4′ as radii, draw the concentric circles required. The radius B C may be divided into any number of equal or unequal parts, which represent the ratio of area contained by the concentric divisions.

FIG. 44.

To inscribe any number of equal circles in a given one.

Divide the given circle A D C into the number of parts that there are to be circles, as 6, and draw radii through two divisions, E and F. Bisect the angle E B F by a line B D; and at D draw a tangent, cutting E and F, and forming a triangle. Bisect the angle B E F by a line cutting B D in a point G the centre of one circle. With G D as radius, draw a circle (as in Fig. 26). Draw a circle from B as centre, with B G as radius. Then the centres of all the circles will lie in this circle, and may be more accurately determined by bisecting each of the divisions of the large circle with radii, cutting the other circles in their true centres. In this instance, the figure is capable of having a circle inscribed in the centre B.

FIG. 45.

To draw a line equal to the circumference of a given circle.

Draw the diameter of the given circle A C, and a perpendicular on the centre B. Draw line A D, the chord of the quadrant, and the versed sine E F, which bisects the angle A B D. Set off on a line, A F, a length equal to three times the diameter A C and once E F. Then line A F will be nearly (that is, as near as can be determined by any simple rule) equal to the circumference of the circle.

FIG. 46.

To find a mean proportional between two given lines.

Let A B and B C be the two given lines. Set off on a straight line A′ B′ and B′ C′, equal to the given lines. Bisect A′ C′ at D. On D as centre, with D A as radius, describe the semicircle A′ E C. At B the junction of the lines draw B E perpendicular to A C, meeting the circle at E. Then B E is the mean proportional required.

NOTE.—When there are three lines, the first of which is to the second in the same proportion as the second is to the third, the second line is said to be a mean proportional between the two others.

FIG. 47.

To find a third proportional to two given lines.

Draw two lines, forming any angle (about 45°), to represent the first and second given lines, A B and C B. Produce the first given line, and make this produced part equal to the second given line; that is, make A′ B′ equal C B, the first given line. Then make A′ B′ and B′ D each equal A B, the second

given line. Join B′ B′; and from D draw D C′, parallel to B′ B′. Then B′ C′ will be the *third proportional less.*

NOTE.—Of these three lines, the first bears the same ratio to the second that the second bears to the third (see mean proportional). If it is required to find a *third proportional greater*,—proceed as before, but reversing the order of the lines; that is, set the first and second lines off from the point of the angle A, one on each line.

FIG. 48.

To find a fourth proportional to three given lines.

Let A C, A B, and C D be the three given lines, and it is required to find a fourth proportional *less.* Draw two lines, forming any angle (about 45°), to represent the first and second given lines, as A′ C′ and A′ B′. Produce A′ C′ (the first given line) to D′, making C′ D′ equal the third given line. Join B′ C′, and draw a line from D parallel to C′ B, to cut A′ B′ produced to E. Then B′ E will be a third proportional *less*, as required. If it is required to find a fourth proportional *greater.*—Set off the two shortest of the given lines from the point A, (one on each line,) as A′ C′, A B. Produce the shortest line, making C′ D′ equal the third and longest given line. Join B′ C′, and draw D E parallel to it, as before. Then B′ E will be a fourth proportional *greater*, as required.

NOTE.—The lines are in the following ratio A′ C′ : A′ B′ : : C′ D : B′ E. In setting off the lines, care must be taken that the first and third given lines are placed together on one side of the angle, and the second and fourth lines together on the other side of the angle.

FIG. 49.

To find a harmonic mean between two given lines; that is, to find a third line, so that the greater of the two given ones is to the less, as the difference between the line found and the greater, is to the difference between this line and the shorter.

Let A B and A C be the two given lines. Mark these lengths together on one line, as A′ B′, A′ C′. On B′ C′, the difference between the two given lines, draw a semicircle, B′ F C′, from D as a centre. On A′ D as a diameter, draw another semicircle, cutting the first at F. Draw a line from F perpendicular to A′ B′, and cutting it at G. Then A′ G will be the harmonic mean between A B and A C. And A′ B′ : A′ C′ : : B′ G : C′ G.

If the arcs of the circles are also marked on the opposite side of the line, the perpendicular may be drawn through the points of their intersection at once. If the two lines A C and A G were given, and it were required to find a *third harmonic progressional* to them, the following construction on the same principle must be adopted. Place the two given lines together as before, and through C draw any line at pleasure (at about 45°), upon it, set off any distance, both ways from C to F and H. Draw two lines, one from A, through F, and the other from H, through G, meeting in F. Through F draw a line F B, parallel to D H, to meet A G, produced to B. Then A B will be the third harmonic progressional to A C and A G.

FIG. 50.

To divide a line medially, or into mean and extreme ratio.

Let A B be the given line. Then on one end of A′ B′ erect a perpendicular A′ C equal to half of A B. Join C B′ and from C as a centre, with C A′ as radius, cut C B′ in D. From B as a centre, with B′ D as radius, cut A′ B′ in E. Then the given line A′ B′ is divided medially in E as required. To produce a given line, A B, so that it shall be one segment of the produced line divided medially.—Divide the given line A B medially as before, and make the produced part equal to B E, the larger segment of this division. Then A B will be the longer segment of the whole line thus produced.

FIG. 51.

Angles of Incidence and Reflection.

A C B represents a plane. The lines D F G E form angles of incidence and reflection with that plane; that is, supposing a ball to be thrown in the direction D C, so as to strike the plane A B in C, it would bound off in the opposite direction C E. Then A C D is the angle of incidence and B C E the angle of reflection, and these two angles are always equal to each other. The angles at which an object is seen and reflected in a piece of water or mirror are the angles of incidence and reflection. Another example is that of the game of billiards.

FIG. 52.

Plane Scales. To make a scale of one foot to an inch, show feet and inches.

Draw a straight line, A B, of any convenient length, and set off equal distances of one inch upon it. To make the points of division more conspicuous, raise small perpendiculars from each of them, A 0, 1, 2. These divisions each represent one foot, and must be marked with figures commencing at the second division, 1, 2, etc., according to the length of the scale. Divide the first inch A 0 into twelve equal parts, for inches, and figure every third inch, as 3, 6, 9, 12, counting the opposite way from 0, to that of the feet. It will be easily seen from this scale, that any dimension can be obtained by one opening of the compasses; thus, for 2 feet 3 inches, place one leg of the compass on 2 B, and the other on 3, (the third division of the inches,) other dimensions being taken in the same manner.

To make a scale of two feet to an inch, to show feet and tenths.

Draw a straight line, C D, as before, and on it set off any number of distances, each equal to half an inch in length. Figure these points, always commencing at the second division 1, 2, 3, etc., to the end of the line. Divide the first division, C 0, into ten equal parts, to represent tenths of feet, and figure the fifth and tenth divisions, counting them the opposite way to that of the feet. If the dimension 3·5 or 3 feet 5 tenths be required, place one leg of the compasses on 3 in the large divisions, and the other leg on 5 in the smaller ones.

To make a scale of one hundred yards to an inch.

Draw a straight line, E F, as before, and upon it set off equal distances, of one inch in length. Figure these points, commencing from the second division, 100, 200, etc. Divide the first 100 yards into 10 equal parts, each to represent 10 yards (this scale being too small to show the units or single

FIG. 41

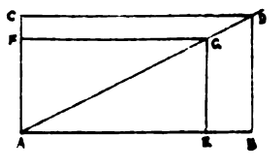


FIG. 42

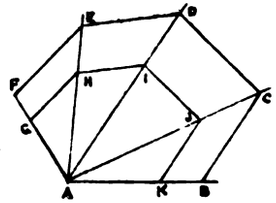


FIG. 43

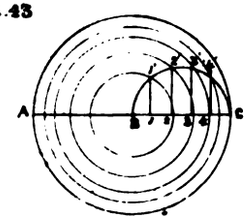


FIG. 44

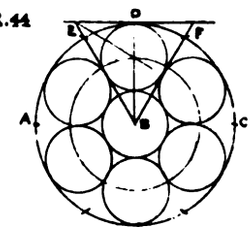


FIG. 45

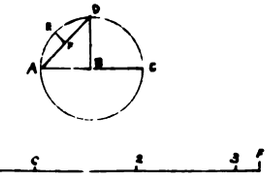


FIG. 46

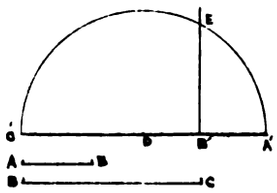


FIG. 47

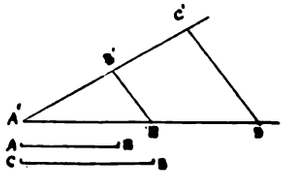


FIG. 48

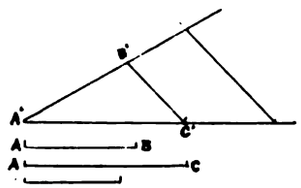


FIG. 49

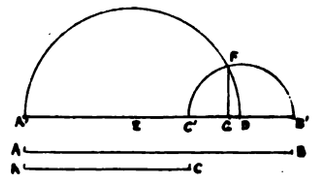


FIG. 50

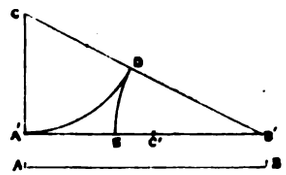


FIG. 51

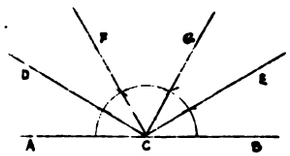


FIG. 52

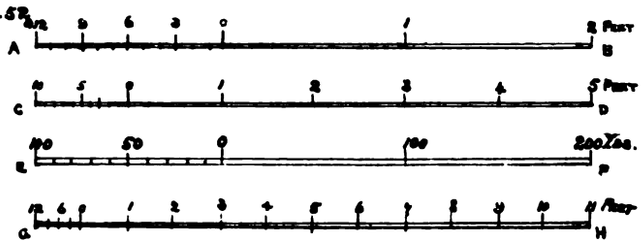


FIG. 53

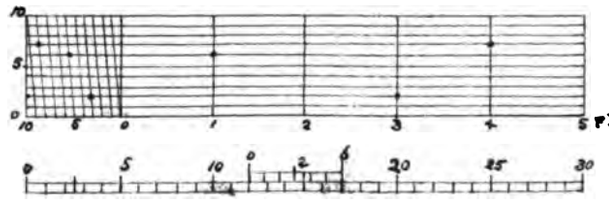


FIG. 54

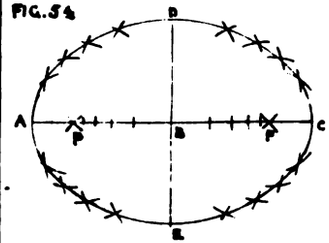


FIG. 55

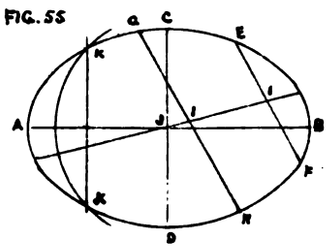


FIG. 56

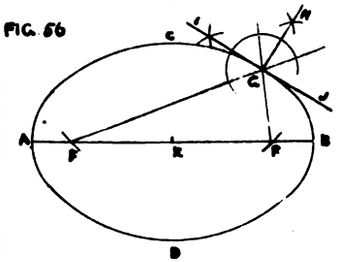
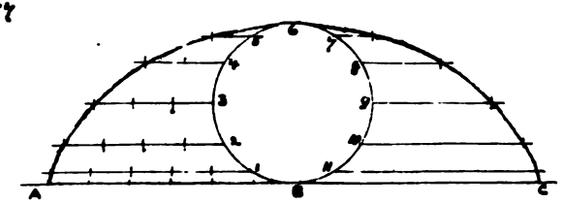


FIG. 57



WILLCOCK & HUDSON'S Geometry.

yards). Figure the fifth of these divisions 50, and the tenth 100. Then 240 yards is measured by placing one leg of the compasses at F and the other to the fourth of the smaller divisions. This and all other small scales should always be divided by diagonal division. (As explained in Fig. 53.)

To make a scale of four feet to an inch, to show feet and inches.

Draw a straight line, G H, as before, and set off on it any number of equal distances, of a quarter of an inch (or one inch distances, afterwards divided into four parts each). Figure the points, G 0, 1, 2, 3, etc. Divide the part of the line G 0 into twelve equal parts, for inches. Figure the sixth and twelfth divisions. The distances are taken from this and all other plane scales in the manner described.

FIG. 53.

Diagonal and Vernier Scales.

To make a diagonal scale of half an inch to a foot, to show feet, tenths of feet, and hundredths of feet, or units, tenths, and hundredths.

Draw any line 10, 5, for a base, and set off a number of equal distances, each half an inch long for feet. Figure these points (counting from the second division) 1, 2, 3, &c. Divide the first part of the line into ten equal parts for tenths. At each of the points 10, 0, 1, 2, 3, &c., draw perpendiculars equal in height to one of these divisions. On the first of these perpendiculars set off any ten equal spaces, and draw ten equidistant lines from these divisions parallel to the base line. Figure the fifth and tenth divisions. Draw ten parallel lines from the divisions of the first foot on the base line, sloping them *one-tenth* of a foot, (or one division) out of perpendicular for *hundredths*, so that each of these lines forms in the height a diagonal to each division of *tenths*. These lines divide each *tenth* again into ten parts at points in the horizontal lines. Thus, for 3·32, that is 3 ft., 3 tenths, 2 hundredths, place one leg of the compasses on the second horizontal line above the figure 3, and the other leg where the third diagonal (of the smaller divisions) cuts the second horizontal. Two other dimensions are shown for examples, marked with dots, namely, 1·56 ft. and 4·87. Both points of the compasses must be in the same horizontal line.

To make a Vernier Scale.

A Vernier (so named from its inventor) is a smaller or auxiliary scale attached to a larger one, to show subdivisions, which is so constructed as to slide backwards and forwards upon the other. Draw any scale of equal parts, as 0, 5, 10, &c., and another for the Vernier, equal in length to so many divisions of the former scale as are one less than the required number of subdivisions ; in this case six is the required number of subdivisions ; the length of the Vernier is therefore made equal to five of the divisions of the larger scale, this number being one less than the former. Divide the length of the Vernier into the required number of equal parts, six, and figure them. It will be evident, that as five divisions of the scale are equal to six divisions of the Vernier, one division of the Vernier will be less than one division of the scale by one-fifth of a division of the scale ; therefore two divisions of the Vernier will be less than two of the scale by two sixths, and so on. Wherever the Vernier is placed or moved to, some of the divisions must coincide with some of the divisions of the scale. The zero, or 0, on the Vernier, is always the index for pointing out the number of divisions of the larger scale ; thus, the zero of the Vernier, as shown, stands at 12 ; but if the Vernier were moved, so that the third division of it was opposite to the fifteenth division of the scale, then the zero of the Vernier would be between the twelfth and thirteenth divisions of the scale ; and it would read thus : 12 and $\frac{3}{6}$th—the third division of the Vernier being the only one that coincided with a division of the scale. Again, if the fourth division of the Vernier was moved opposite to the twenty-third division of the scale, it would read thus : 19 and $\frac{4}{6}$ths, the zero of the Vernier being in advance of the nineteenth division by four sixths.

FIG. 54.

To describe an ellipse, the length and breadth (or transverse and conjugate diameters) being given.

Bisect the *transverse* axis A C at B, and through B, draw the perpendicular D E, making B D and B E each equal to half the *conjugate* diameter. On D and E as centres, with B A as radius, describe arcs cutting at F, F′ for the foci or centres. Divide F B into any number of parts at random ; those nearest to F F′ should be made smaller, so as to determine more points in the curve towards A and C. Make the divisions in each half of the ellipse alike. On F F′ as centres, with a radius equal to the distance from A to the first division, draw arcs. Again, on F F′ as centres, with a radius equal to the remainder of the line (from the first division to C), draw arcs, cutting the last and giving four points in the curve. Repeat this operation, taking the distances from A to the second division, and from thence to the end of the line C as radii, from F F′ as centres, the arcs, cutting in four more points of the curve. Next take the distances from A to the third division, and so on. Draw a curved line through all these points by hand.

NOTE.—The curve of an ellipse can be drawn mechanically by means of *Trammels*[1], as follows :—Draw the two diameters A C and D E. Find A F, the difference between these diameters. Set the points of the Trammels to A and F, (the difference,) and the pencil marker, or third point, to B (half the shorter diameter). Turn the Trammels round, always keeping the two fixed points, one in each of the diameters.

FIG. 55.

The curve of an ellipse being given, to find the two diameters.

Draw two parallel lines, in any direction, across the curve, as E F and G H. Find the middle of each of these lines, and mark them I I. Draw a line through the points I I, and mark the middle of this line J. From J as a centre, draw any arc cutting the ellipse at K K. Draw a line through K K, and

[1] Trammels consist of a long bar with two or more sliding points, adjustable at any part of the bar by a screw.

another, C D, parallel to it and passing through J. Then a line A B, drawn through J perpendicular to C D, completes the figure. A B and C D being the two diameters required.

FIG. 56.

To draw a tangent and perpendicular to an ellipse at any given point, as G.

Find the centres, F F, of the ellipse by the foregoing rules. Join the centres F F with G, and produce these lines. Bisect the angle formed by the produced parts, then G H is a perpendicular to the curve. At G, bisect the angle formed by the lines F G and F G produced. Then I J will be a tangent to the curve at G, and also a perpendicular to G H.

FIG. 57.

To draw a cycloid, the generating circle being given.

The *cycloid* is a curved line traced by a point in the periphery of a circle during one revolution on a straight line.

Let B 6 be the generating circle. Draw a line A B C equal to the circumference of the generating circle, by dividing the circle into any number of parts, as 1 2 3 4—12, and marking half that number of parts off from B to A, and from B to C. Draw lines from the divisions of the circles—1 2 3 4 5, and 7 8 9 10 11, parallel to A C. Set off one division of the circle outwards on the first lines 5 and 7; mark off two divisions on the second lines 4 and 8, then three on the next, and so on. Draw the curve by hand, passing through the ends of these lines, and the cycloid is found.

FIG. 58.

To draw a curtate cycloid, the generating circle and position of the generating point being given.

The *curtate cycloid* is a curve traced by a point outside a circle, making one revolution on a straight line.

Draw the generating circle A B, and the generating point C, at the distance C B from the circle, and placed on the diameter 4 B produced. Draw a base line F F', touching the circle, and equal to it in circumference; the diameter bisecting this line. Divide the circle into any number of equal parts, 1 2 3 4, &c., and draw the radii. Draw lines from each of the divisions of the circle and on each side of it, parallel to the base line F F'; also one G G' at the distance C B, from the base line and below it. Divide the base line F F' from 0 to 8 into the same number of equal parts as the circle. On each of the lines that join the radii, set off distances from the circle in the following manner:—Take one division of the circumference, and mark it on the first line at each side of the circle (as 1', 1'; 7', 1'). Mark two on the second line (2', 2'; 6', 2'). Mark three on the third, and so on. From the ends of all these lines, 1 2 3 0, and 1 2 3 8, draw lines 1 D, 2 E, 3 F, 4 G, etc., parallel to the nearest radial line. On each of these last-drawn lines, and from the ends of the others, mark off a distance 1 D, 2 E, 3 F, 4 G, etc., equal to C B, the distance of the point from the circle. Then the curve line G' F' E' D', C D E F G, drawn through those points, is called a curtate cycloid, as required.

FIG. 59.

To draw a prolate cycloid, the generating circle and point being given.

If a circle be supposed to make one revolution on a straight line, the curve traced by any point within the circle, is called a prolate or *inflected cycloid.* The construction of this figure is similar to the previous one. Draw the generating circle A B, and the point C within it. Draw the base line D 8 equal to the circumference of the circle. Divide the circle into any number of equal parts, and draw the radii, 1 2 3 4, etc. Draw lines from the divisions of the circle on each side of it, and parallel to the base line. Mark off a distance on the first line equal to one division of the circle. Mark off a distance on the second equal to two divisions; also on the third equal to three divisions, and so on. From the ends of the lines joining the radii, draw lines towards the circle, parallel to the nearest radii. Mark off a distance on each of these lines equal to B C (the distance of the point from the circle). Draw a curved line through the last-found points G F E D, C D E F G, which completes the figure.

FIG. 60.

To draw an interior epicycloid, the two circles being given.

If one circle be supposed to revolve within, and upon the circumference of another, the curve line traced by any point in the circumference of the rolling circle during one revolution is called the *interior epicycloid.* Two of these curves are shown in the figure formed by two revolutions of the circle. Draw the larger circle, A B being the radius, and the smaller one, B C, touching at B. Divide the smaller or generating circle into any number of equal parts, as in this case 8. Set off distances, 1 2 3 4, etc., round the circumference of the larger circle, each equal to the divisions of the smaller one. Draw radial lines from each of these points to the centre A of the larger circle. From A as a centre, draw circles passing through each of the points of division on the smaller circle; that is, taking for radii in succession the distances A B, A 1, A 2, A 3, and A 4. Draw another circle, passing through C, the centre of the rolling circle, and mark the points *c' c' c'*, where it is cut by the radial lines. Then, regarding these points as the position of the centre C during the revolution of the circle, from each of these points, *c' c' c'*, etc. with B C as radius, draw arcs cutting the circles alternately at 5 5 5, etc. and giving so many points in the curve. Draw a curved line through these points from B to 8, which will be the interior epicycloid required.

FIG. 61.

To draw an exterior epicycloid, the diameters of the two circles being given.

The construction in this figure is similar to the previous one, the only difference being, that the rolling circle is outside the fixed one.

Draw the two given circles, A B and B C, touching at B. Divide the smaller or rolling circle into any convenient number of equal parts, and set off distances equal to these divisions round the largest circle at 1, 2, 3, 4,

FIG. 58

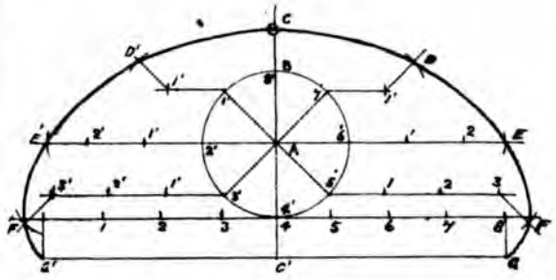


FIG. 59

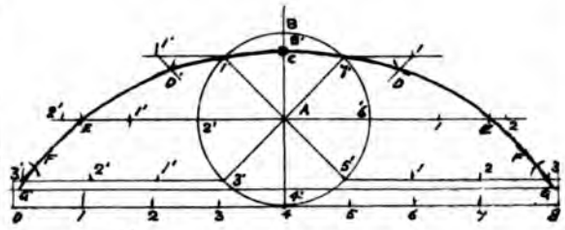


FIG. 60

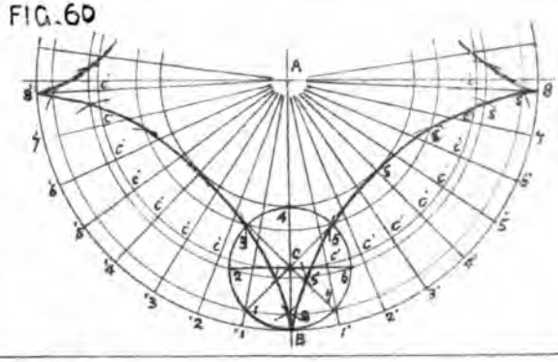


FIG. 61

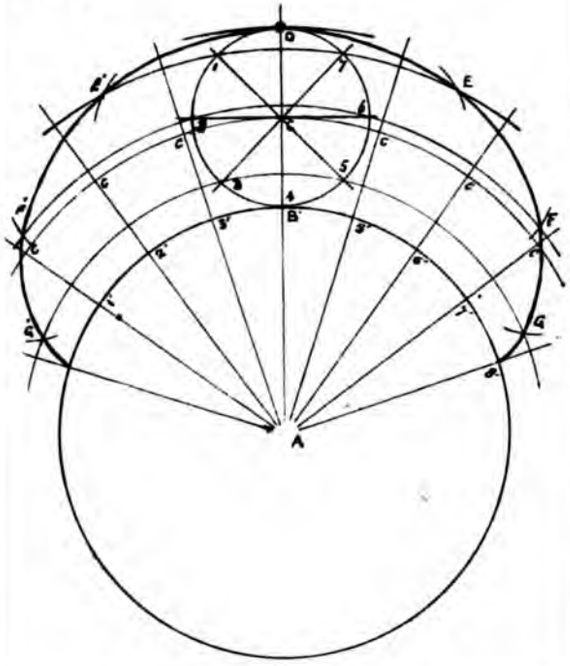


FIG. 62

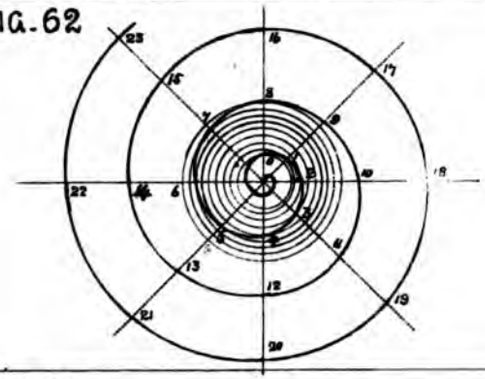


FIG. 64

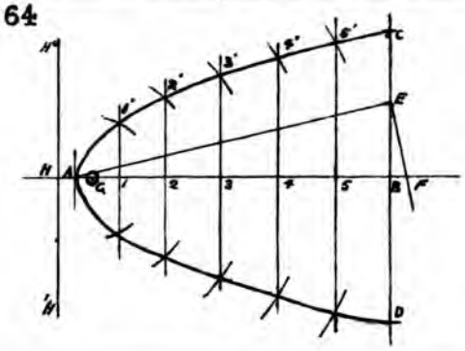
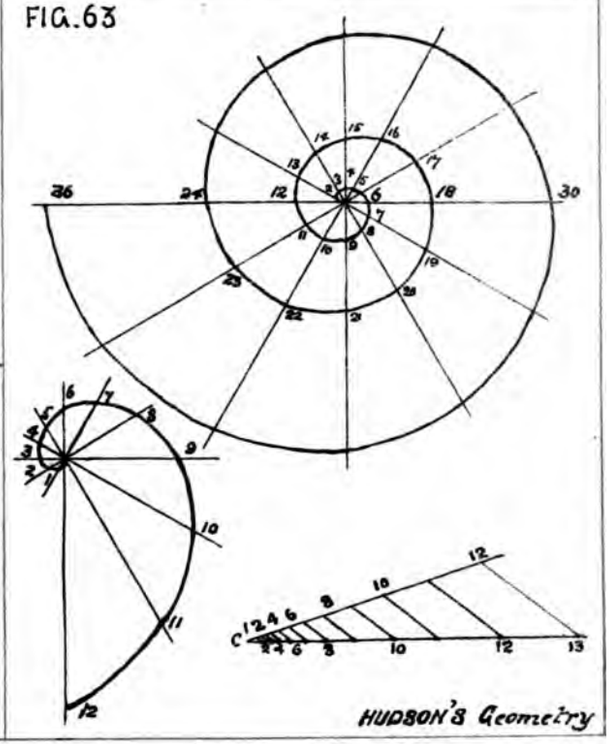


FIG. 63



etc. Draw radial lines from A through those points. Draw circles from A as centre, passing through each of the divisions of the rolling circle, and cutting the radials at *c c c*. Taking *c c c*, etc., as centres, and with B C as radius, draw arcs, to cut the concentric circles alternately at E F G, etc. Draw a curved line through G F E D, E′ F′ G′, which will be the curve required.

FIG. 62.

To draw an Archimedian spiral or equi-distant scroll.

Draw a circle to represent the size of the first spiral, the radius being equal to the distance between each turn or spiral. Divide the circle into any convenient number of equal parts, and draw radial lines through these divisions. Divide the distance between one turn into the same number of equal parts as there are radial divisions, and draw concentric circles 0 1 2 3, etc. Mark the point where the first circle cuts the first radial, then where the second circle cuts the second radial, and so on. Draw a curved line through these points, 0 1 2 3, etc., thereby completing one turn or revolution. The second and third turns may be afterwards found without repeating this process, by making the distance between the turns equal, and parallel throughout, thus: mark the width 0 8 from 1 to 9, from 2 to 10, from 3 to 11, and so on. Every alternate point in the spiral will increase or diminish one unit from the centre. If the spiral is to consist of a given number of turns, and to pass through a given point, A, opposite to the other end next the centre B.—Divide the distance A B into the same number of equal parts as there are to be turns. Draw radii as before, and subdivide each of the parts on A B into the same number as there are radii. Then set off the length of one of the subdivisions inwards, on the first radial next to the point A, set off two on the second, three on the third, and so forth. The spiral must be drawn through these points by hand, and the scroll must finish at the centre C opposite to B.

This spiral may be drawn by winding a string on or off any cylinder, a pencil or tracer being fixed to the end of the string, so as to mark its course.

FIG. 63.

To draw a logarithmic spiral.

In a separate figure draw a scale of ordinates in continued proportion, which is done as follows:—Draw two lines, C 12 and C 13, forming any angle (about 45°), and set off two very short unequal distances, one on each line, C 1 and C 2. Find a third proportional, C 3, to them; and, by the same rule, successively find lines C 4, C 5, C 6, etc., in continued proportion to the lines previously obtained. Draw (in a separate figure) any number of radial lines making equal angles with each other, which is done by drawing a circle and dividing its circumference into any number of equal parts, and drawing lines through the divisions. Commence upon any of the radii, and mark off a distance 0 1, from the centre outwards, equal to C 1 (the first division of the scale of ordinates). Upon the next radial, set off a distance 0 2, equal to C 2 (the two first divisions of the same scale). Upon the third radial, set off a distance 0 3 equal to C 3 (the three first divisions of the scale); and so on to the twelfth or last radial. Connect the points thus found on the several radii by a continued curved line, which forms the first complete revolution of the spiral, as shown enlarged. To find the second and third revolution of the spiral, commence again with the first radial line, and upon it set off a distance equal to C 13 (the thirteen first divisions of the same scale produced). On the second radial, set off the fourteen first divisions, and so on, going round and round. Draw the continuation of the curve through these points, which will continually increase in its distance from the centre, as the square of the distance.

The continuation of the figure, according to the above rule, is shown on a smaller scale, in order to suit the size of the book.

FIG. 64.

An abscis, or portion of the axis of a parabola, and its ordinate, or width, being given; to find the focus, and to draw the curve, by means of intersecting arcs.

When any cone is cut by a plane parallel to one of its sides, the section is called a parabola.

Draw two lines at right angles to each other, A B the given "axis," and B C the "ordinate." Bisect the ordinate B C in E. Join A E, and draw a perpendicular to it at E, meeting A B produced, to F. From the "vertex" of the parabola A, set off a distance upon the axis from A to G, equal to the produced part of B F. Then G is the "focus" of the parabola. Draw a line H′ H″, called the directrix, at the same distance from A as A is from G. Produce the given ordinate C B to D. Make B D equal B C. Divide the given axis into any number of parts, 1 2 3 4 5. In the parabola, as in the ellipse, the outline of the figure is very much curved near the vertex; whilst it is so little curved in the other parts that portions of it almost approach to a straight line; therefore, it is advantageous to divide the axis unequally, making the divisions smaller towards the vertex A. Draw lines through each of the divisions 1 2 3 4 5, parallel to the double ordinate C D. From G as a centre, and with H I, draw an arc to cut the first division line on both sides of the axis, as at I′. From G again as a centre, with H 2 as radius, cut the second division line, as at 2′. From G, with H 3, cut the third line at 3′, and so forth, always measuring the radii from H, and drawing it from G as a centre; because every point in the curve is equi-distant from the focus and directrix. Connect all the points of the intersection by a curved line from A to C, also from A to D. This curve is the parabola required, and the double ordinate C D is its base.

NOTE.—A rectangle to the parabola in area may be made by taking two-thirds of its length, A B, for one side, and its double ordinate, C D, for the other.

FIG. 65.

To divide a triangle into any number of equal or proportional parts by lines drawn parallel to one side of it.

Let A B C be the given triangle. Divide one of its sides, B C, into the same number of equal or unequal parts, as the triangle is to have parts, and mark them 1 2 3. On the divided side, B C, as a diameter, draw

a semicircle 1′ 2′ 3′, and from each, point off divisions 1 2 3. Raise perpendiculars, cutting the semicircle in 1′ 2′ 3′. From the vertex C of the triangle, as a centre, with radii C1′ C2′ and C3′, draw arcs from where the perpendiculars meet the semicircle to cut the side B C. Then, from these points where the arcs cut the side of the triangle, draw lines 1″ 2″ 3″ parallel to the base A B, which, in this instance, divides the given triangle into four equal parts.

NOTE.—The triangle may be divided by lines drawn parallel to either of the sides by this rule. It will be observed that the side which the lines are drawn parallel to, is termed the "base," and the opposite corner the "vertex." Any side except the "base" may be divided, but the "vertex" must always be used as a centre for the arcs.

FIG. 66.

To divide a triangle into any number of equal or proportional parts, by lines drawn from a point in one of its sides.

Let A B C be a given triangle, and D a point in one of its sides. Divide that side which contains the given point into any number of parts, as B′ 1 2 3 4 C. Draw a line from the point D to the opposite angle A, and from each of the divisions 1 2 3 4 on B C, draw lines parallel to it (D A), to cut the opposite sides of the triangle. Join the point D with the points 1′ 2′ 3′ 4′, where the parallels cut the sides; and these lines will divide the triangle into the required number of parts, having their areas in the same ratio as the divisions of the side.

FIG. 67.

To divide a triangle into any number of equal or proportional parts by lines drawn from one angle of it.

Let A B C be a given triangle, and C the angle from which the lines are to be drawn. Divide the base A B (or side opposite to the angle C) into the required number of equal or proportionate parts, as in this instance, six in the points 1 2 3 4 5. Draw lines from these points to the angle C, which will divide the triangle into the required number of parts.

FIG. 68.

To divide a triangle into three equal parts, by lines drawn from the angles to a point within it.

Let A B C be the given triangle. Bisect any two sides, as A B, and B C, in 2 1, and from each of these points draw lines to the opposite angles C and A. Where these lines cut each other in D, draw a third line to the remaining angle. Then the thick lines meeting in D, will divide the triangle into three equal parts.

FIG. 69.

To divide a lunule into any number of equal or proportional parts.

Draw an arc, and a chord of it, A B; upon A B as a diameter, describe a semicircle; then the space enclosed between the arc and semicircle, will represent the given lunule A 1′ 2′ B. Divide the chord A B in the required ratio, 1 2. At each point of division, raise a perpendicular, cutting the upper or exterior arc in 1′ 2′. From these points draw lines towards the centre of the lower arc D. The radiating lines thus drawn will divide the given lunule in the manner required. In this case the lunule is divided into three equal parts.

FIG. 70.

To divide an ellipse into any number of equal sectors.

Draw an ellipse, as A 1′ 2′ B 4′ 5′; and upon the longest axis, A B, as a diameter, draw a circle. Divide the circumference of the circle into the same number of parts as the ellipse. From the points of division 1 2 4 5, draw ordinates or perpendiculars to A B, cutting the ellipse in 1′ 2′ 4′ 5′. From the centre of the ellipse and circle C, draw radii to the points of intersection (1′ 2′ 4′ 5′) on the circumference of the ellipse. These lines will divide the given ellipse into the required number of sectors or parts.

An ellipse may also be divided into any number of equal concentric parts, on the same principle as the circle. See Fig. 43.

FIG. 71.

To divide any rectilineal figure into equal or proportional parts, by lines drawn from a given angle of it.

Let A B C D be a figure to be divided, and E the angle to which the lines are to be drawn. Reduce the figure to an equal triangle by Fig. 92, having its vertex in the given point E. A 5 being the base of the equal triangle, divide it into the required number of equal or proportional parts, as A 1 2 3 4 5. Produce the side B C; and draw lines from the point E to each of the angles, as E B and E C. From those divisions, which are in the side A B, draw lines to E. Draw lines from the divisions 2 3 4 5, parallel to E B, cutting the side B C produced, in 2′ 3′ 4′ 5′. Join E 2′ and E 3′. Then draw lines from 4′ and 5′, parallel to the radial E C, cutting the side of the figure C D in 4″ and D. Join E 4″, which is the last division line in this case; but if the figure has more sides, continue this process, producing the next side, and drawing lines from the divisions on it, parallel to the next radial line, to cut the next side of the figure.

FIG. 72.

To divide any rectilineal figure into equal or proportional parts by lines drawn from a certain point within it.

Let *a b c d e f* be any given figure, and *g* the point from which the division lines are to be drawn. Reduce the figure to an equal triangle, as in the previous figure, by Fig. 92. Then change this obtained triangle into an equal one, having its vertex in the point *g* (and its base in all produced) by Fig. 79. Divide the base, A B, of the triangle into the same number of parts, whether equal or proportional, as those into which the given figure is proposed to be divided, as 1 2 3 4 5 6 B. Join any division which falls on the side *a b* of the figure, as 4, with the point *g*. Draw radials from the point *g* to each of the angles *a b c d e f*. Produce the sides. Draw lines from each of the divisions 1 2 3, on A B, parallel to the radial *a g*, to cut the sides *a f*, or *a f* produced, as 1, 1′, 2, 2′, 3, 3′. Join 3′ *g*. Produce the side *f e*, and draw lines from 1′ and 2′ parallel to the radial *f g*, to cut the side *f e* or *f e* produced. Join 2 *g* and *e g*. Produce the side *f e*, and draw lines from 1′ and 2′, parallel to the radial *f g*, to cut the side *f e* or *f e* produced. Join 2 *g* and *e g*. Produce the side *b c*, and draw lines from the divisions 5 6 B, parallel to the

Plate 6.

FIG. 65

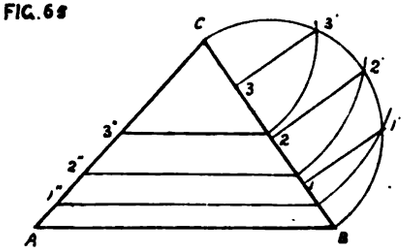


FIG. 66

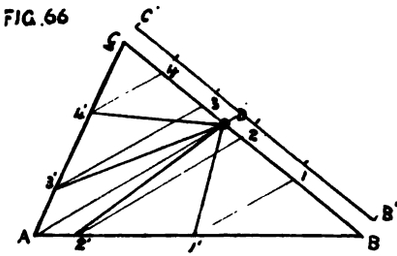


FIG. 73

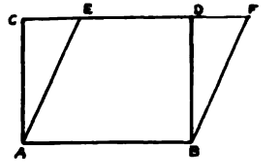


FIG. 74

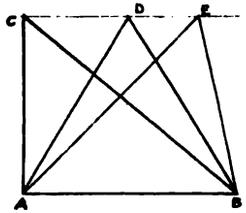


FIG. 75

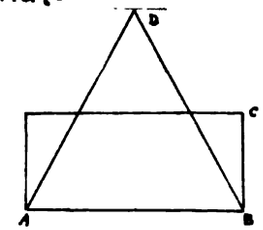


FIG. 67

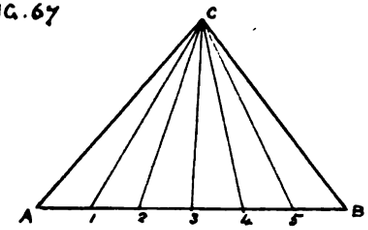


FIG. 68

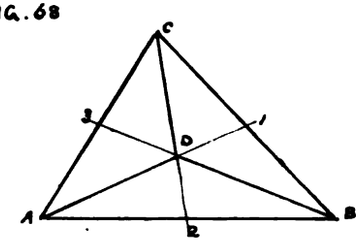


FIG. 76

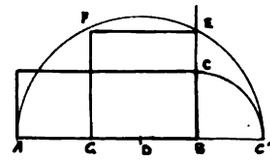


FIG. 77

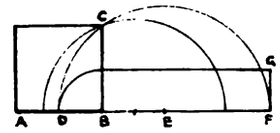


FIG. 78

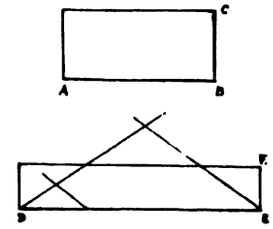


FIG. 69

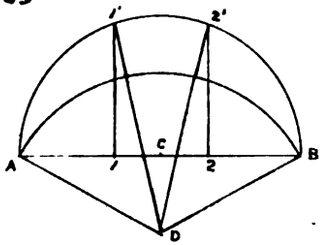


FIG. 70

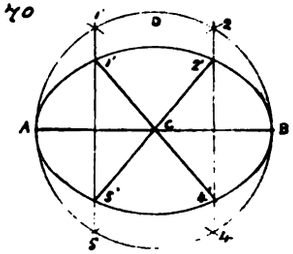


FIG. 79

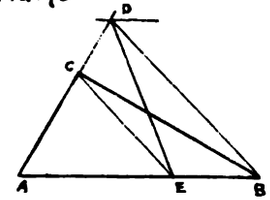


FIG. 80

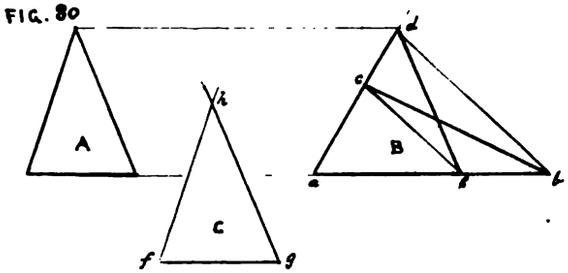


FIG. 71

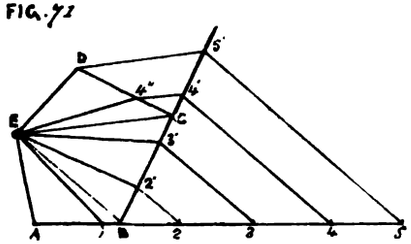


FIG. 72

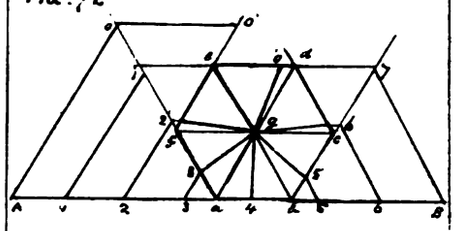


FIG. 81

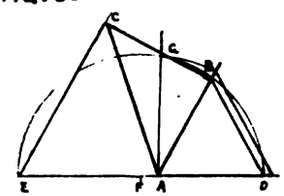
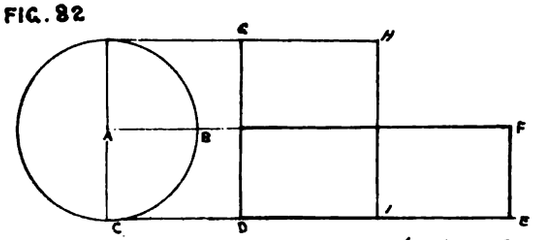


FIG. 82



HUDSON'S Geometry.

radial *b g*, to cut the side *b c* or *b c* produced. Join B 5. Produce the side *c d*, and draw lines from 6 and 7, parallel to *c g*, cutting *c d* or *c d* produced. Join *g* 6 and *g* 7. But if, as in this case, the last division falls beyond the side *c d*, draw a line from the point on the side *c d*, parallel to the next radial, *d g*, to cut the side *d e* in *d'*, and join *d' g*. Then these lines *g o*, *g* 1, *g* 2, *g* 3, *g* 4, *g* 5, and *g* 6, will divide the figure *a b c d e f* into the required parts or areas.

FIG. 73.

All parallelograms, whether right or oblique angled, having equal bases and equal altitudes, have equal areas.

A B C D is a right-angled parallelogram or rectangle. A B E F is an oblique-angled parallelogram. The base A B and perpendicular height B D is common, or the same for both; therefore A B C D is equal to A B E F. The superficial areas of all parallelograms are obtained by multiplying their base by their height. *All parallelograms having equal altitudes have to each other the same ratios (or proportion) that their bases have. Again, all parallelograms having equal bases have to each other the same ratios that their altitudes have.*

FIG. 74.

All triangles having equal bases and equal altitudes, have equal areas.

The right-angled triangle A B C has the same base and altitude as triangle A B D, and also triangle A B E; therefore, their areas are equal. The superficial areas of all triangles are obtained by multiplying their base by half their altitude (or perpendicular height), or their altitude by half the base.

All triangles having equal altitudes, are to each other in the same ratio as their bases; or, having equal bases, are in the same ratio as their altitudes.

FIG. 75.

To draw a triangle equal in area to a rectangle; or to draw a rectangle equal to a given triangle.

Let A B C be a given rectangle. Make any triangle on A B having twice B C for its perpendicular height, as A B D. Then triangle A B D will be equal in area to the given rectangle A B C. If the triangle A B D be given, take half its perpendicular height for the height B C of the rectangle, and make the base of the rectangle equal to that of the triangle. Then the rectangle will be equal in area to the given triangle.

NOTE.—Therefore, any triangle drawn on the base of a rectangle, and having twice the perpendicular height, or having the same height and twice the base, is equal to it in area, and *vice versâ*. Any rectangle having the same base as a triangle and half the height, or half the base and the whole height, is equal to it in area.

FIG. 76.

To draw a square equal in area to any given rectangle.

Let A B C be a given rectangle. Produce A B to C', making B C' equal B C. On D, as a centre, draw a semicircle A F E C'. Produce the side B C to cut the semicircle E. Then B E will be a mean proportional, as in Figure 46, between the two sides of the given rectangle (A B and B C), on which construct a square B E F G.

FIG. 77.

To draw any rectangle equal in area to a given square.

Let A B C be a given square. Produce the side A B indefinitely, and determine the length of one side of the required rectangle, as B F. On E, as a centre, with a suitable radius, draw a semicircle to pass through F and C, cutting the base-line in D. Draw the rectangle B F G, making F G equal to B D. Then B F G will be the required rectangle, equal in area to the given square. In the same way, if B D be given to find B F, draw the semicircle through D C to cut A B produced in F.

The second side of the rectangle may be obtained by Fig. 47, finding a third proportional between the given side of the rectangle and square.

FIG. 78.

To draw any rectangle equal in area to a given one, but having a different base or height.

Let A B C be a given rectangle, and D E the base of the required one. Find E F, a fourth proportional (Figure 48) to A B, B C, and D E. In the following manner, draw a line from D, making any angle (about 45°) with D E. Mark off a distance on it equal to A B. On D E mark off a distance equal to B C, from D towards E. Join the ends of the two lines, and draw a line parallel to it from the intermediate point in D E, to cut the other line at a distance from D, equal to the height of the required rectangle, as E F. Then the rectangle D E F will be equal to the given rectangle A B C. If the side E F is given to find D E, proceed as before; reversing the process to find D E, the fourth proportional greater.

FIG. 79.

To draw a triangle equal to a given one, but having a different base or altitude.

Let A B C be the given triangle, and D the altitude of the required one. Produce one side, as A C, to cut line D. Join D B, and draw a line parallel to it from C to cut the base A B in E. Draw the diagonal D E, which will be the side of the triangle required. Then triangle A E D will be equal to triangle A B C. If the base, A E, is given to find the height D, join C E and draw a line parallel to it from B, to cut the produced side A C in D. Draw line E D as before. This is only the reverse of the foregoing.

NOTE.—This figure may also be worked by a fourth proportional to the height and base of the given triangle, and the height or base of the required one, as in the previous figure.

FIG. 80.

To draw a triangle equal in area to a given one, and similar to another.

Let *a b c* be the given triangle, representing the given area; and A the form of the required triangle. Change triangle *a b c* into one (*d e d*) of the same height as A by the previous figure. Find a mean proportional, *f g*, between the base of the triangle *a b d*, and triangle A. On *f g*, as a base, construct a triangle similar to A. Then this triangle, C, will be equal to one, B, and similar to the other, A. A triangle may be made similar to a given one, and equal to any given figure or area by this method.

FIG. 81.

To draw an equilateral triangle equal in area to a given triangle not equilateral.

Let A B C be the given triangle. Draw an equilateral triangle upon one side of the given triangle, as A B D. Produce line D A. From C draw a line parallel to the side A B, and cutting the produced line, D A, in E. By Fig. 46 find a mean proportional, A G, between lines D A and A E; that is, describe a semicircle upon D E. At A, draw A G perpendicular to D E, and cutting the semicircle in G. Line A G will be the mean proportional and side of the required equilateral triangle, equal in area to the given triangle. Therefore, any parallelogram, polygon, circle, or trapezium, may be converted into an *equilateral triangle* having the same area, by first changing the given figure into any equal triangle, and then proceeding as described.

FIG. 82.

To draw a rectangle equal in area to a circle.

Let A be the centre of the given circle. Draw a diameter through A, and also a radius A B, perpendicular to it, so as to find a quarter of the circumference B C. Draw a rectangle D G H I, making one side, D G, equal the diameter of the circle, and the other side, G H, equal B C, one quarter of its circumference. Or, make a rectangle D E F, having one side, E F, equal to the radius, A C, of the circle, and the other side, D E, equal to one half of its circumference. Then each of these rectangles, D G H I and D E F, are equal to the given circle. Therefore, a triangle may also be made equal to a circle, by making its height equal to the diameter, and its base equal to half the circumference.

FIG. 83.

To draw a triangle of any given area.

Draw a rectangle, A B C D, to represent the given area. This is done by making one side, A B, one unit in length, and the other side the same number of units or parts as the area. Thus, for an area of three and a half units, make A B 1 unit and B C $3\frac{1}{2}$ units. In some cases, it is found more convenient to take two or four units for its height and half or a quarter the area in units for its base. Erect any perpendicular, as E F, equal to twice the height of the rectangle; and join E B and E C. Then the triangle, B E C, will be equal to the given rectangle, and hence contain the given area. Therefore, a triangle may be made to contain any given area at once without drawing the rectangle, by making its base equal and its height double.

FIG. 84.

To draw a square of any given area.

Make a rectangle A B C D, to represent the area, as in the previous figure. Find a mean proportional (*c e*) between the sides of the rectangle, B C and C D. Then a square made on *c e* for its side, will contain the area required.

FIG. 85.

To draw a rectangle equal to a given square, when one side of the rectangle is given.

Let A B C be the given square. Draw the given side of the rectangle, B E, on A B produced; then the other side may be found by drawing a semicircle through E and C to cut A B, giving B D; or, by finding a third proportional to the side of the square and the given side of the rectangle.

FIG. 86.

The area of a circle is equal to a rectangle, when two of the sides are equal to half the circumference, and the other two to the radius of the circle.

If a circle, A B, be divided into a number of equal parts or sectors, as twenty-four, and twelve of them are opened out upon a straight line, A B, which will be equal to half the circumference (nearly)—the remaining twelve parts, being inverted, will, with the others, form a rectangle nearly equal to the circle (the error is about $\frac{1}{1800}$).

FIG. 87.

To draw a triangle equal in area to a given sector.

Let A B C D be a given sector. Find a rectangle, B C A, equal to the sector, by making C A equal the radius, and B C equal half the circumference of the sector. Then a triangle having its perpendicular height the same as, and its base double that of, the rectangle, will be equal to the given sector.

FIG. 88.

See Fig. 79.

FIG. 89.

See Fig. 81.

FIG. 90.

See Fig. 80.

FIG. 91.

To draw a triangle equal to the sum of two or more given triangles.

Let A and B be the two given dissimilar triangles. Draw a triangle, *a b c*, similar to B. Produce the side *a c* to *d*. Change this triangle into one of the same height as A by Fig. 79—*d e* being the side of the new triangle (*a d e*). Add the base of the triangle A to *a c*, which produces it to *f*. Join *d f*. Then the triangle *a d f* will be equal to the sum of the two triangles. If there are more triangles to be added, change each into triangles of the same height as the first, and add their bases together.

FIG. 92.

To reduce any given figure to an equal triangle.

Let A B C D E be the given figure, and the vertex of the triangle be required at D. Then join A C, and draw a line from B parallel to it, cutting the base produced to *a*. If *a* and C are joined, *a* C D E will be equal to the given figure. Join *a* D, and draw a line from C parallel to it, cutting the base produced to *f*. Join *f* D, then the triangle *f* D E will be equal to the given figure as required. And if the given figure has more sides, proceed in the same way, reducing the figure one side less each time, until it becomes a triangle. The vertex of the triangle may be in any of the corners or sides, and the base may be a continuation of any side.

FIG. 93.

To draw a triangle equal to a given lunule.

Let A B be the given lunule. Join A B, the diameter of the outer arc, and find the diameter D E of the inner arc. From D as a centre, with the diameter of the outer arc, A B, as radius, cut the inner arc in

FIG. 83

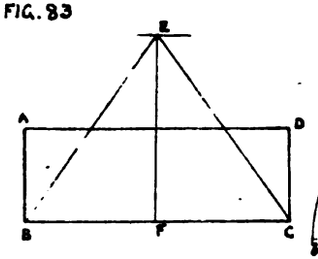


FIG. 84.

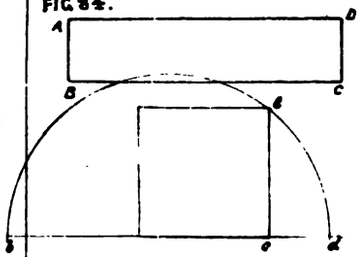


FIG. 85

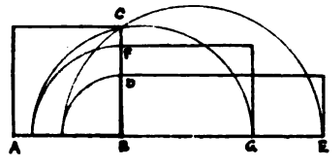


FIG. 86

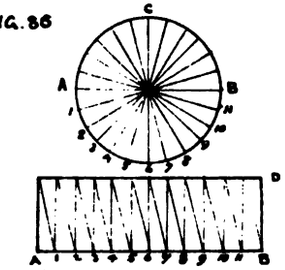


FIG. 87

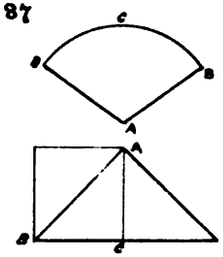


FIG. 88

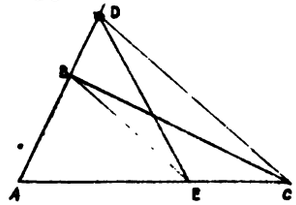


FIG. 89

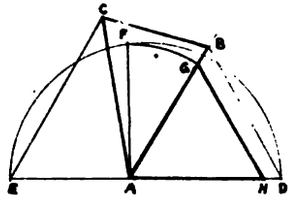


FIG. 90

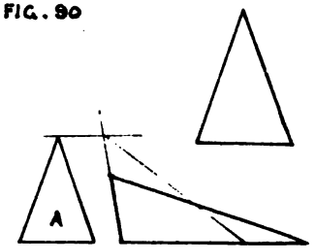


FIG. 91

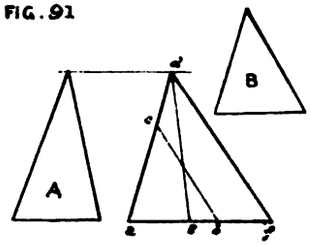


FIG. 92

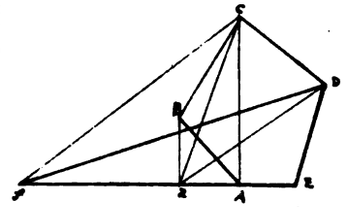


FIG. 93

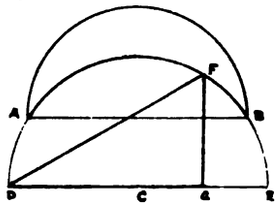


FIG. 94

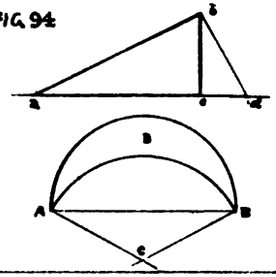


FIG. 95

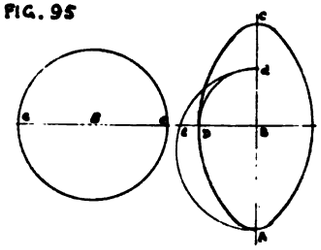


FIG. 96

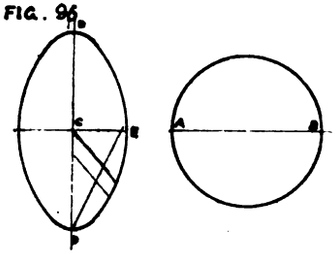


FIG. 97

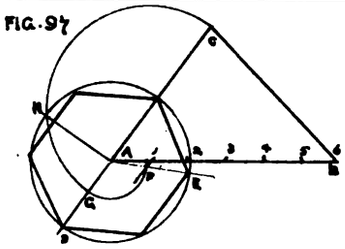


FIG. 98

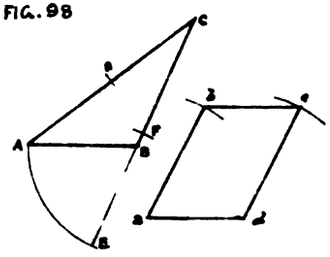


FIG. 99

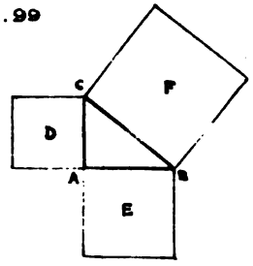


FIG. 100

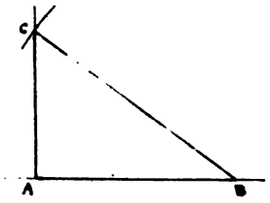
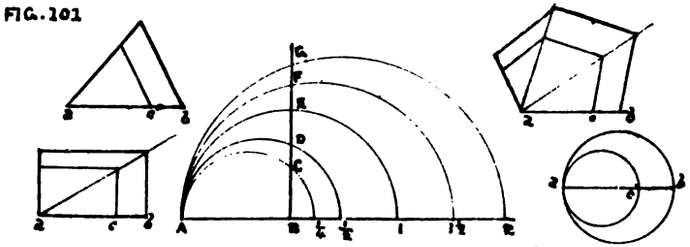


FIG. 101



F. Join D F, and from F to G draw a line perpendicular to D E. Then the triangle D F G is equal to the given lunule.

FIG. 94.

To draw a lunule equal to a given triangle.

Let *a b c* be the given triangle, which must have one right angle. Draw a perpendicular to the longest side *a b* at *b*, cutting the other side produced in *d*. Draw a semicircle A B, with half *a b* as radius, then this will be the outer arc of the lunule. Take half *a d* as radius; and from a point C, draw an arc, cutting the other at A and B. This being the inner arc. The space D enclosed by the two circles forms the lunule which is equal to the given triangle.

FIG. 95.

To draw a circle equal to a given ellipse.

Let A C be the given ellipse. Find a mean proportional between A B and B D; that is, add these lines together, and draw a semicircle A *e d*. Then a perpendicular at B cuts the semicircle at *e*, and B *e* is the radius of the required circle *e* B *e*, equal in area to the ellipse.

FIG. 96.

To draw an ellipse equal to a given circle.

Let A B be the given circle. Determine one axis or diameter of the ellipse, as D D. By Fig. 47 find a third proportional to half A B and half D D. Then C E, the third proportional, will be half the other diameter of the ellipse required; and the curve being drawn by the rule of Fig. 54, this ellipse will be equal in area to the given circle. It is obvious that either the half or the whole diameters may be taken to find the third proportional.

FIG. 97.

To draw any regular polygon equal to a given triangle.

Let A B C be any given triangle. Divide one side of the triangle (as A B) into the same number of equal parts as there are to be sides to the polygon—in this case, six. Draw a line A E, forming an angle with the other side A C, of one-sixth of 360°, which is the angle at the centre of the required polygon. From the first division 1, draw a line parallel to A C, to cut A E in F. Find a mean proportional (A H) between A C and A F. Then A H will be the radius of the circumscribing circle, D E H, of the polygon. Produce A F to E, then A C and A E cut the circle into one-sixth. Draw the chord of this part, which will form one side of the polygon. Complete the polygon by stepping this distance round the circle, and joining the points.

FIG. 98.

To draw a parallelogram equal in area and perimeter to any given triangle.

Let A B C be the given triangle. Draw two indefinite parallel lines, *a d* and *b c*, at a distance from each other equal to the perpendicular height of the triangle from the base A C to the point B. Take half A C for the base of the parallelogram *a d*. From *a* and *d* as centres, with a radius E F equal to half the sum of the other two sides of the triangle, draw arcs cutting the line in *b* and *c*. Join *a b* and *a c*. Then this parallelogram, *a b c d*, is equal to the triangle in area, because it has the same perpendicular height, and half the base of the triangle. The perimeter or sum of the sides is equal to that of the triangle, because the two sides, *a d* and *b c*, are together equal to the base A C, and the two sides *a b* and *d c* are equal to the sum of the two other sides A B, B C.

FIG. 99.

To draw a square equal to the sum of two or more given squares.

Let A B and A C be the sides of two given squares D and E. Place these two sides at right angles to each other. Join the ends C and B. Then the square F, on B C, is equal to the squares on the other lines.

This rule is useful in adding any other similar figures together.

FIG. 100.

To draw a square equal to the difference between two given squares.

Let A B and B C be the sides of the two given squares. Draw A B, and at one end of it erect a perpendicular, A C. From B, with a radius B C, draw an arc to cut the perpendicular in C. Then A C is the side of a square equal in area to the difference between the two given ones. The difference between any two similar figures may be found by this means.

FIG. 101.

To draw any figure having an area in any proportion, greater or less, than a given similar figure.

Let A B represent the base or side of a triangle or parallelogram, or the diameter of a circle or polygon.

1st. If the proposed figure be a triangle, *a b* being its base, and it is required to find a triangle of one-half its area. Produce A B indefinitely. Take half of A B, and set it off from B, as B$\frac{1}{2}$. Draw a semicircle on the whole line A$\frac{1}{2}$. At B erect a perpendicular, cutting the circle at D. Then B D will be the base of the new triangle, *a c*, required, shown by the line at *c*, drawn parallel to the side of the given triangle (so as to retain the same angles).

2nd. If the given figure is a parallelogram or rectangle, *a b* being one side, and a similar figure of one quarter the area be required. Draw A B, equal to *a b*, and add one quarter of A B to it; and on A$\frac{1}{4}$ as a diameter, draw a semicircle, cutting the perpendicular at B in C. Then B C is the side of the required rectangle *a c*. Draw the diagonal of the rectangle, and also a line from *c* parallel to the second side of it. From the point where this side cuts the diagonal, draw another line parallel to the third side of the given figure. Then this inner figure will be similar, and contain one quarter the area of the outer one.

3rd. If the given figure be any irregular polygon, and it is proposed to draw one similar, of one half the area, proceed as before. From A draw lines passing through all the angles, and from the point found, *c*, draw lines parallel to the sides of the figure, and cutting the radii, as shown.

4th. If the given figure is a circle or a regular polygon, of which *a b* is the diameter, let A B represent this diameter. It is proposed to draw a figure of 1$\frac{1}{2}$ times the area of the given one. From B set off 1$\frac{1}{2}$ times A B (as at 1$\frac{1}{2}$). On A 1$\frac{1}{2}$, as a diameter, describe a semicircle. Erect a perpendicular

at B, cutting the semicircle at F. Then B F will be the diameter of the required circle or polygon. In this case, an error often occurs by adding one half A B, instead of one and a half times A B. This process is, in fact, finding a mean proportional between the side or diameter of the given figure, and a line representing the proportion of area of the similar figure. As, for example, a figure is to be constructed similar to a given one, but having twice the area. Then a mean proportional between the side of the given figure, and a line twice that length, will be the side of the required one.

FIG. 102.

To divide a given line, internally or externally, so that the rectangle made of its two segments shall be equal to a given rectangle.

It must be understood that if the given line be shorter than twice the length of a mean proportional between the two sides of the given rectangle, it will be impossible to divide it into two parts by *internal* divisions, in which case the line must be divided *externally* or artificially.

Method 1.—Let *a b c* be the given rectangle, and *e f* the given line. At each end of the given line (*e f*) erect a perpendicular, and make one perpendicular (*f h*) equal to the base, and the other (*e g*) equal to the height of the given rectangle (*a b c*). Join the ends of these perpendiculars by a line, *g h*. Bisect it in the point *i*, from which as a centre, with a radius equal to half the line (*g h*), draw an arc of a circle to cut the given line, as at *j*. This intersection will divide the given line into two parts as required, and a rectangle made of these parts, as *f j k l*, will be equal in area to the given rectangle, *a b c*. But, should the proportion of the line to the rectangle be such that the arc will not cut the given line, proceed according to the following method, which is practicable in all cases.

Method 2.—Let *a b c* be the given rectangle, and A B the given line. At each end of the given line (A B), but on contrary sides of it, draw a perpendicular. Make one perpendicular equal to the base, and the other equal to the height of the rectangle (*a b c.*) Join the ends of the perpendiculars by a line C D. Bisect it; and from the centre E, with a radius equal to half the line C D, draw an arc to cut the given line produced, in F. Then a rectangle made on the whole line A F for its base, and having the produced part B F for its height, will be equal to the given rectangle *a b c.*

Note.—The given line may also be divided so that the rectangle of its parts shall be equal to any given figure, if the figure first be converted into an equal rectangle.

FIG. 103.

To divide a given line, so that the squares on the parts shall have any given ratio of area to each other.

Let A *b* be the given line. Then draw a line *a b*, making an angle (about 45°) with it. Set off along *a b*, from the point *b*, a distance to *c*, and another to *d*, to represent the ratio of the squares, which is done by laying down distances equal to so many units taken from any scale. Draw a square on *b c*, as *c b e f*, and a rectangle on *c d* of the same height as the square. Change the rectangle *d c f g* into a square of equal area, as *a c h i* (by Fig. 76). Join *a* A, and draw a line parallel to it from *c*, to cut the given line A *b* in B. Then A *b* will be divided as required; and the squares drawn on these parts, A B and B *b*, will be to each other in the required ratio, which is represented by the squares *c b e f* and *a c h i*.

FIG. 104.

To divide a given straight line into two parts, so that the rectangle contained by the whole line and lesser part, shall be equal to the square on the other.

Let A B be the given line. On A B as a side, draw a square A B D C. Bisect A C in E, and join E B. Produce C A to F, making E F equal to E B. On A F draw a square A F H G; or set off A F on A B to G. Then A B will be divided in G, so that the rectangle on the whole line and lesser part (G B D I) is equal to the square on the other part, A G (A F H G), as required.

FIG. 105.

Theorem 1.—*If, in a circle, two straight lines cut one another, the rectangle contained by the segments of one, is equal to that contained by the segments of the other.*

Example.—Let A B and E G be two lines or chords cutting one another at F. Then a rectangle made with the two segments F A and F B of the first chord, will be equal to the rectangle made with the two segments F E and F G of the second.

Theorem 2.—*If any two chords of a circle not cutting each other within the circle, be produced until they meet in a point; the rectangle made with the whole line and exterior segment of the one, is equal to the rectangle made with the whole line and exterior segment of the other.*

Example.—Let E G and *j i* be two chords produced till they meet in D. Then a rectangle, made with D E and D G for its sides, will be equal to the rectangle made with D *j* and D *i*.

Theorem 3.—*If two lines be drawn from any point outside a circle, one of which lines passes through the circle, and the other touches it; the rectangle made with the whole line which cuts the circle, and the part outside the circle as sides is equal to the square made with the tangent as a side.*

Example.—Let D E be the line cutting the circle, and D *h*, the tangent or line to touch the circle, both lines being drawn from the same point, D. Then a rectangle made with D E and D G for its sides, will be equal to the square on D *h*; that is, with D *h* for its side, *h* being the exact point of contact with the circle.

FIG. 106.

From a point in one side of any given rectilinear figure, to draw a line cutting off from it a segment equal to a given triangle or other figure not greater than the former.

Let A B C be the given triangle representing the area of the part to be cut off by the line; and let E D F G H I J be the given figure, with K for the given point. By Fig. 79, change the triangle A B C into one of the same height as E K, and mark off

FIG.102.

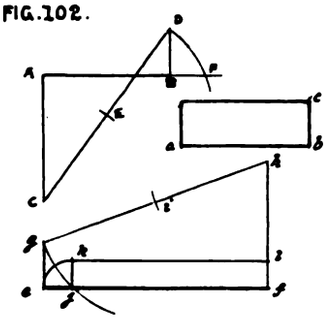


FIG.103.

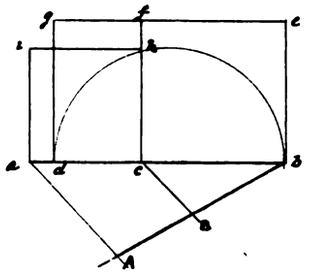


FIG.104.

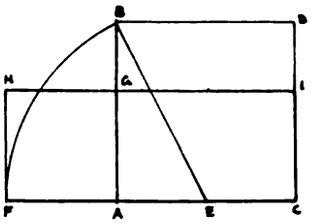


FIG.105.

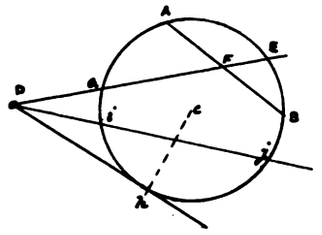


FIG.106

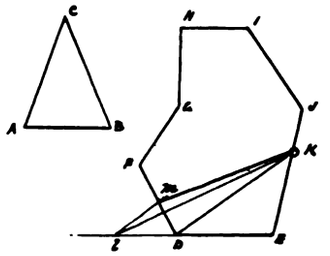


FIG.107

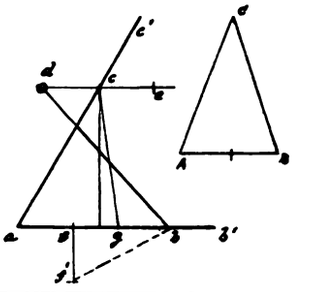


FIG.108.

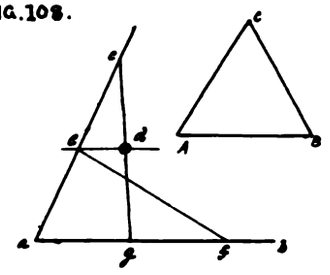


FIG.109

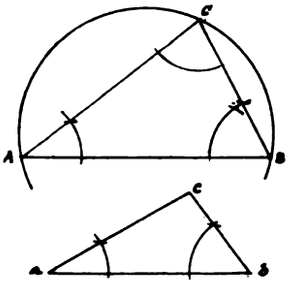


FIG.110

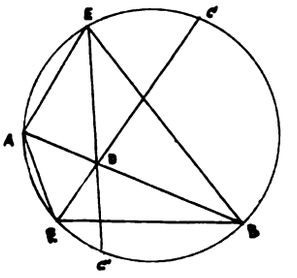


FIG.111

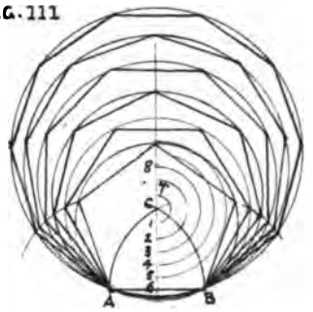


FIG.112

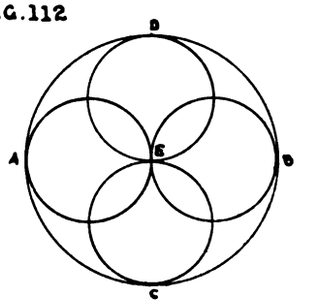


FIG.113

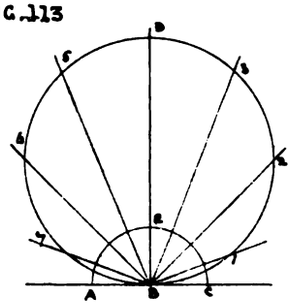
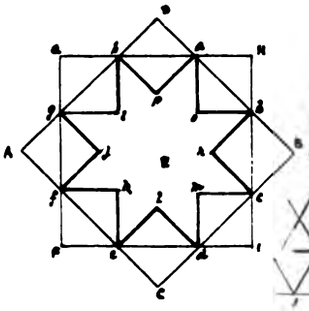


FIG.114



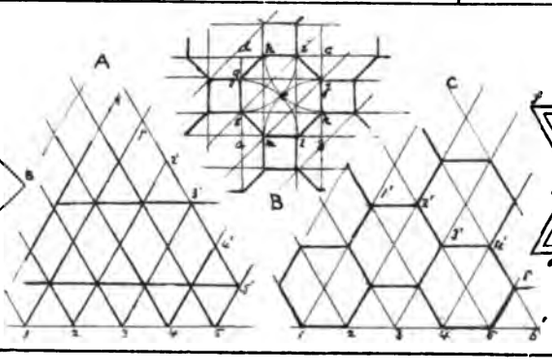
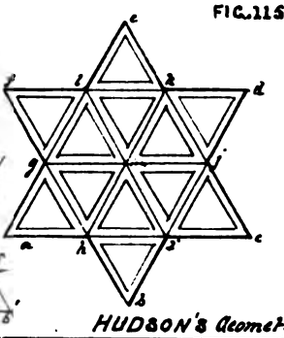


FIG.115



the length of its base from E to *l;* or, in other words, make a triangle K E *l*, equal to the given triangle A B C. If the base of the triangle found be less than the side of the figure E D, join its extremity with the point K; then this side of the triangle will be the line required. But if the base of the triangle found be greater than the side E D, as E *l*,—draw a line from the angle D to the point K, and another parallel to it from *l* to cut D F in *m*. Join *m* K. Then the line *m* K will cut off a portion, K *m* D E, of the figure, equal to the given triangle. If the line *l m* does not cut D F, as will often occur—proceed as before; that is, produce D F, and draw a line from the next angle F, to K. From the point in D F produced, where the line from *l* cuts, draw a line parallel to F K, to cut F G in a point, say *n;* then join *n k*, which will be the line required. This process is similar to that in Fig. 71.

FIG. 107.

The two sides of an angle and a point outside of them being given, to draw a line from the said point, to cut off a portion of the angle equal in area to a given triangle.

Let *a b'* and *a c'* be the lines forming the given angle, and *d* the point outside. Draw a triangle A B C to represent the area of the part to be enclosed or cut off. From the point *d*, draw a line parallel to one side of the given angle to meet the other in C. For the vertex, make a triangle, *a c g*, equal to the given A B C. Bisect the base of this triangle *a g* in *f*. Produce *d c* to *e*, making the produced part *c e* equal to *a f*. At *f*, drop a perpendicular *f f'* equal to *c d'*. From *f* as a centre, and with *d e* as radius, cut the side *a b'* in *b*. Then a line drawn from *b* to the point *d* will cut off a portion of the given angle equal to the triangle required.

FIG. 108.

The two sides of an angle, and a point within them, being given—to draw a line through the said point to form, with the two given lines, a triangle equal to a given one.

Let *a b* and *a c* be the two sides of the given angle, and *d* the given point. From the point *d*, draw a line parallel to one side of the given angle to meet the other side in *e*. On *a b* as a base, with *e* for the vertex, make a triangle *a e f* equal to the given one A B C. By Fig. 79, produce *e d* on the other side of the point. Bisect *a f* in *g*. Mark off a distance to *h* on *e d*, from *e* equal to *a g*.

Case 1.—If the distance *e d* be equal to the produced part *d h* of the same line, then a line drawn from *g* through the point *d*, and cutting the other side in *c*, will complete the figure.

Case 2.—If the distance *e d* be greater than the produced part *d h* of the same line, the problem will become impossible.

Case 3.—If the distance *e d* be less than the produced part *d h* of the same line, proceed as in the previous figure; that is, drop a perpendicular at *g* equal to *e d*. From the extremity of this perpendicular, as a centre, with a radius equal to the produced part *d h* (of the line *e d*), cut the base line *a b*, and from this point draw a line passing through the point *d*. This line will be the one required.

FIG. 109.

The segment of a circle (or the base and opposite angle of a triangle) being given, to draw a triangle, having two of its sides in a given ratio.

Let A B C be the given segment of a circle, which will contain the given angle (see Fig. 25), and A B the base of the triangle. A C B being the given angle. Draw any triangle *a b c*, having one angle (*a c b*) equal to the given one A C B. Make the two sides containing the angle to each other in the required ratio; that is, make *c b* to *c a* in any given ratio, and join the ends of these sides by a third line *a b*. At A and B, make angles equal to those found at *a* and *b*. Produce the sides of the angles to cut each other at C. If the arc is given or drawn, it will only be necessary to copy one of the angles at the base.

FIG. 110.

From any two points in the circumference of a circle, to draw two lines to meet in a third point (in it), which lines shall be to each other in any given ratio.

Let A and B be the two given points in the circle. Join A B, and bisect the arc in C'. Divide the line A B in the given ratio, as in D'. Draw a line from C' through D, cutting the circle in E, the point required. Draw lines A E and B E; then these lines will be in the required ratio. In the figure, two points are shown, to which lines may be drawn; one point E is found by bisecting the arc A C' B, and the other by bisecting the opposite arc A C B. Therefore, there are two points in the circumference to which lines may be drawn, having a given ratio.

FIG. 111.

To construct any number of regular polygons upon a given side.

Let A B be the given side. Bisect A B, and draw a perpendicular C. From A as a centre, with A B as radius, draw an arc B C. Divide the arc (B C) into six equal parts. From C, as a centre, draw circles passing through the divisions on the arc B C, and cutting the perpendicular in the points 1 2 3, etc. These points (1 2 3, etc.) will be the centres of the polygons and their circumscribing circles. C is the centre of a six sided-polygon. From the point C, draw a circle through A and B. Step the distance, A B, round this circle, and join the points, forming a hexagon.

For a five-sided polygon, from the point 1 as a centre, draw a circle through A B, and step A B round the circle as before, but forming a pentagon (or five-sided figure).

For any polygon having more than six sides, set up from C as many parts of the arc B C as, added to six, make the required number of sides. Thus, for a seven-sided polygon, set up one; for an eight-sided, two; for a nine-sided, three; and so on. Then these points in the perpendicular are the centres of the polygons; and circles drawn through A and B, give the size of each polygon.

FIG. 112.

Ratios of circles.

If within a given circle, A B C D, four others be inscribed, having their diameters each equal to the radius of the given one, the area of these four circles will together be equal to that of the given one. There-

fore, a circle having its diameter equal to the radius of another, will contain one quarter the area. The areas of circles are to each other as the squares of their diameters.

FIG. 113.

To divide the circumference of a circle by means of angles set off from a tangent.

If a circle and tangent be drawn (B D and A C), and any semicircle (A E C) be divided into a number of equal or proportional parts to represent the ratios of the required divisions of the circle, lines drawn from the point B through the divisions on the semicircle, will cut the circle in the same proportion. Or, if any number of equal or unequal angles be set off on the tangent, and lines drawn from B to cut the circle, the circle will be divided into parts bearing the same ratios as the angles.

FIGS. 114 and 115.

Construction of various Geometrical Patterns, used in tesselated pavements and ornamental designs.

FIG. 114

Is a star of eight points, drawn by means of two squares, with their centres common to each other; the sides of one parallel to the diagonals of the other. The points *i j k*, etc., found by the intersection of lines drawn from *a b c d*, etc., parallel to the sides of the squares.

FIG. A

Is a figure showing the method of drawing a pattern composed of equilateral triangles and diamonds, or of triangles or diamonds only.

Mark off the width of the triangles on a line 1 2 3, etc., and draw parallel lines at 60° in each direction. Then draw the necessary lines to form the required pattern, and colour the alternate spaces.

FIG. B

Is a figure showing the method of drawing a pattern composed of octagons and squares. Draw a square *a b c d*, equal to the size of the octagons. Draw two diagonals to find the centre. From each corner of this square, as a centre, draw an arc, cutting the centre *e*, and the sides in *f g h i j k l m*. Join these points, forming an octagon. To multiply the pattern, draw lines through the corners of the octagon in each direction parallel to the sides, and join the intersections. Darken the lines forming the octagons and squares.

FIG. C

Is a figure or pattern composed of hexagons, and is constructed in the same way as figure A. Each hexagon contains six equilateral triangles, and the side of the hexagon is equal to that of the triangle.

FIG. 115

Is a star of six points, filled in with triangles. This figure is drawn by means of two equilateral triangles, having their centres common to each other, and the sides of one parallel to the sides of the other consecutively. The interior triangles may be filled in at pleasure.

PRACTICAL SOLID GEOMETRY.

THIS division of Geometry relates to the theory of Mechanical and Architectural Drawing, and treats of the representation of points, lines, planes, and solids, in their various positions—determined according to their projections. It includes Orthographic and Isometrical Projections; Linear, Parallel, and Oblique Perspective; also, the Projection of Shadows and Construction of the Envelopes of Solids.

ORTHOGRAPHIC PROJECTION.

ORTHOGRAPHIC Projection is simply the representation of figures, as lines, planes, and solids, in plan and elevation, in fixed positions as viewed from any given point.

This description of drawing is unlike perspective, insomuch as the lines are all drawn parallel instead of diverging to vanishing points.

To define or determine the position of any line, plane, or solid, it is necessary to refer to two fixed planes at right angles to each other, called the *vertical* and *horizontal planes*.

All the construction lines and auxiliary projections are shown in fine lines. These lines may be rubbed out when the figures are completed.

FIG. 116.

No. 1 shows an end elevation of the two planes. 5, B, is the *horizontal plane*, and 5, 6, the *vertical plane*. No. 2 is a front elevation of the two planes, A 1, 2, 5, and A 5, which is also the ground line of each figure. All the elevations are shown above the ground line A B, and all the plans below. Therefore, the ground line serves both to represent the *horizontal plane* in the elevations, and the *vertical plane* in the plans.

No. 3.—A 5, 3, 4, represents the *horizontal plane* in the plan, and A 5, the *vertical plane*.

A line, *a b*, is shown in a certain vertical position in No. 1; its front or second elevation at right angles to the first, is shown in No. 2; and its plan at *a*, in No. 3. The height of the line *a b* is the same in each elevation, and its plan is a point at the same distance from *a* to *b*, No. 3, as from *b* to 5 No. 1, projected by the arc *b b'* and line *b' a*.

FIG. 117.

No. 1 shows two lines, *a b*, and *b c*, at right angles to each other; then *a b*, No. 2, is the second elevation of these lines, and *a b*, No. 3, the plan of them.

NOTE.—The two planes of projection are shown here; but in future, figures will be omitted, the ground line A B being sufficient, when the positions and objects of these planes are understood.

FIG. 118.

To represent, by plan and elevation, a line one inch long, inclined to the horizontal plane at 50°, *and also to the vertical plane at* 60°.

Draw a line one inch long, *a b*, No. 2, inclined at 50° to the ground line (A B), and find its plan by dropping a perpendicular from *b* to 2. At any convenient point on A B (1), as a centre, with 1, *a'* and 1, 2, as radii, draw quarter circles. Then *a b* (No. 3) is the width of the line in the plan, and *a b* (No. 1) is the second elevation of the line, if required. Transfer the plan of the line *a b* (No. 3) to *a b* (No. 4), placing it at the other angle (60°) to the vertical plane represented by A B for the plan. Draw up two lines from *a* and *b*, perpendicular to A B, one cutting the ground line at *a*, and the other drawn indefinitely. From *b* (No. 2) draw a line parallel to A B, cutting the former line in *b* (No. 5). Join *a b* (No. 5). Then No. 4 and No. 5 are the plan and elevation required.

FIG. 119.

To represent by plan and elevation two lines, each one inch long, which meet at right angles, when both are equally inclined and lying in a plane inclined at 55°.

Draw the two lines *a b* and *b c* (No. 1), which form a right angle, and equally inclined (each being at 45° to A B). Draw an end view of these lines, as *b' c'* (No. 2). Turn this line on *b'* as a centre, until it forms an angle of 55°, which gives the end view of the lines, as lying in this plane, and also the data for constructing the plan and elevation. From *c* (No. 2), draw an indefinite line parallel to A B, giving the height of the extremities of the given lines. The width *a c* in No. 1 is not affected by the lines being turned into the plane. Drop perpendiculars to A B from *a* and *c*, cutting the horizontal line in *a'* and *c*. Join *b a'* and *b c'* with thick lines. The angle of these lines in the elevation is 110°.

The plan is readily obtained as follows: Produce the lines *a a'* and *c c'* below the ground line; transfer

the width *b' c* (No. 2) on A B to the plan, as shown by the arcs drawn from any point (1) as a centre. Draw the centre line *b* No. 1, *b* No. 3; also a line from 2 and *c* (No. 3) to *b* and *a*. Then *a b* and *b c* joined will be the plan of the lines, and *b a*, *b c*, the elevation required. The angle contained by these lines in the plan is 108°.

NOTE.—The conditions required in this and similar figures are: 1st. The length of the lines. 2nd. The original or real angle contained by them. 3rd. The apparent angles contained by them in the plan. 4th. The same in the elevation. 5th. Their measurements of width and height in the elevation. 6th. The inclination of the plane in which they lie; and 7th. The inclination of each line.

The 1st and 2nd conditions determine the form of the figure to be represented; the others are for determining the position only; and any two of the latter are sufficient to find the remainder. Hence this figure may be constructed from various conditions; the following is an example which will easily be understood from the previous description. The lines each being one inch long, and meeting at right angles, are equally inclined; and in this position contain an angle of 110° in the elevation. From which may be determined the plan and elevation of these lines, the angle of the plane in which they lie, and the angle contained by them in the plan.

FIG. 120.

To draw the plan and elevation of two lines meeting at right angles, when one is inclined at 60° *and the other at* 23°.

When the lines are indefinite or of infinite length, a certain portion or point in them must be taken, and afterwards the lines can be produced.

Draw a line, say one inch long, *a c* No. 1, inclined at 23° to the ground line; also another, *a b* No. 2, inclined at 60°; and a third line forming a right angle with the last, as *a c*. From *c'* No. 1, draw a line parallel to A B, cutting *a c* No. 2 in *c*, and giving the length of the line in the elevation.

Because in No. 2, when *a b* is drawn parallel to the vertical plane, *a c* revolves in a plane at right angles to *a b*.

Then *a b*, *a c* is an elevation of the lines, as lying in the required position.

From *b a* and *c* No. 2, drop perpendiculars to the ground line, and produce them into the plan. Then *a b* No. 3, drawn parallel to the ground line, will be the plan of *a b* No. 2.

Take *a c''* No. 1 (which will be the length of the other line for the plan) and from *a* No. 3 draw an arc cutting the perpendicular *c c*. Join *a c*, completing the plan. These lines contain an angle of 140° in the plan.

NOTE.—The height and angle or direction of a line is always sufficient to determine its length in the plan and elevation. The length and height of a line in the elevation determines its inclination. The angle and length of a line in the elevation determines its height. Inclined is always understood to mean inclined to the horizon or horizontal plane, unless otherwise mentioned. Inclined to the vertical plane or ground line, means lying oblique, or forming an angle with the observer, some times called the plane of the elevation.

FIG. 121.

To draw the plan and elevation of three lines which meet in a point, each at right angles to the plane of the other two, and when these lines are all equally inclined.

The lines being equally inclined to each other, and to the horizontal plane, form equal angles in the plan No. 1.

Mark off a distance on the ground line, as at *a* 1, No. 2, equal to one of the lines, and erect a perpendicular at *a*. Place the other two lines (or such parts of them as are equal to the first) equally inclined at 45°, as *a c*, *a d*, No. 3. Draw a parallel to the ground line from *c*, to cut the perpendicular in 2, No. 2. Then *a* 2 is the height or end view of these lines. Join 1 2. Turn the lines *a* 1, *a* 2, upon the point *a*, until the line 1 2, joining the points of the lines, becomes horizontal. Or draw a line *a* 3, perpendicular to 1 2, and another line *a b*, making an angle with *a* 2, equal to the angle 1 *a* 3; and draw *a c*, equal to *a* 2, at right angles to *a b*. Drop perpendiculars from *a b* and *c* extended into the plan. Then *a b* (No. 1) drawn parallel to A B and *a c*, *a d* drawn, so as to form equal angles, completes the plan No. 1.

A second elevation, No. 3, is easily made, by drawing the three lines, *a b*, *a c*, and *a d*, the same height as those in No. 2; *a c* and *a d* being drawn between the vertical parallels, so as to preserve the same width.

When the three lines are not equally inclined, proceed as in Fig. 133, regarding the lines as three edges of a cube, or four-sided pyramid.

FIG. 122.

To draw the elevation of a square of one inch side, when its plane (or face) is inclined at any angle as 50° *to the vertical plane (or plane of the elevation).*

No. 1 and No. 2 are the common plan and front elevation of the square. Draw the plan 2 3, No. 3, making an angle of 50° with the ground line. Then lines drawn up into the elevation from 2 and 3, give the width in that view; and the height being made the same as No. 2, completes the elevation required. If the elevation (No. 4) is given, the angle in the plan is easily found by drawing down the lines, and placing the side of the square 2 3 between these parallels.

FIG. 123.

To draw the plan and elevation of any triangle, when its face (or plane) is inclined, or set oblique to the vertical plane at any angle, as 45°.

The description of the previous figure applies equally to this, and all similar figures.

FIG. 124.

To draw the plan and elevation of any pentagon, with its face (or plane) inclined to the vertical plane at any angle, as 45°.

This is only another example of Fig. 122, and will be readily understood without a repetition of the description.

FIG. 125.

To draw the plan and elevation of any triangle, with its face (or plane) inclined at any angle, as 30°.

Draw the common view of the triangle *a b c* (No. 1), giving its position and measurements. Above the

FIG. 116.

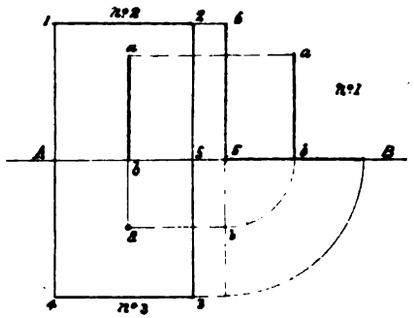


FIG. 117

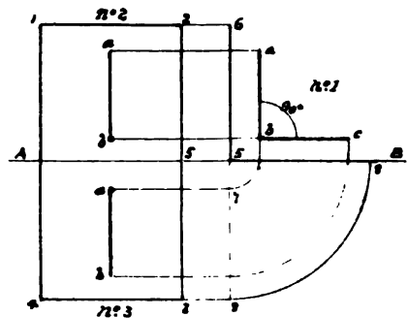


FIG. 118

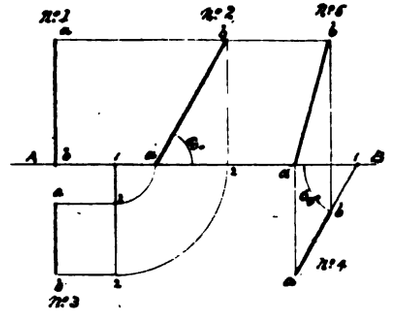


FIG. 119

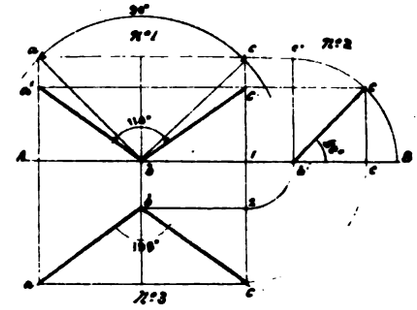


FIG. 120.

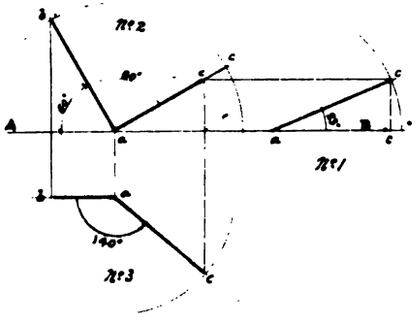


FIG. 121

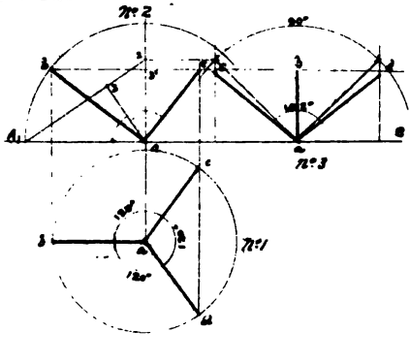


FIG. 122.

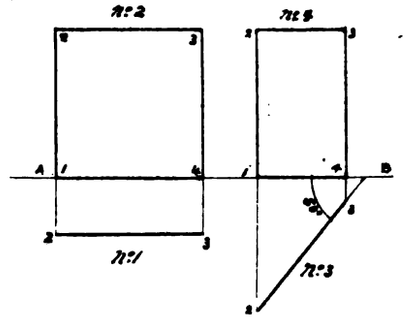


FIG. 123

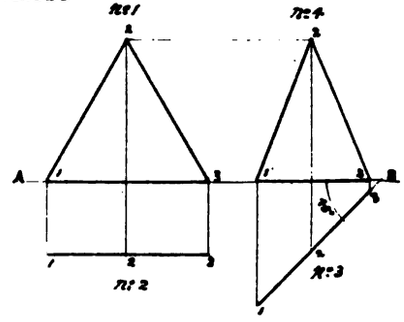


FIG. 124

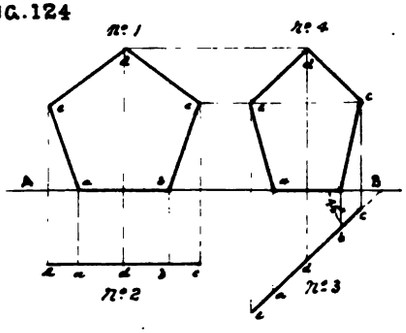


FIG. 125

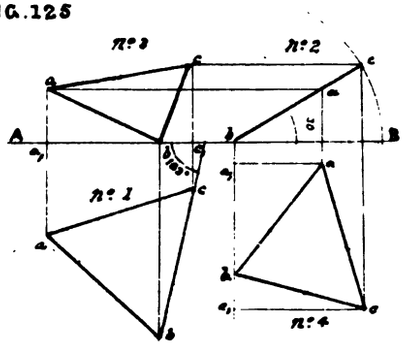


FIG. 126.

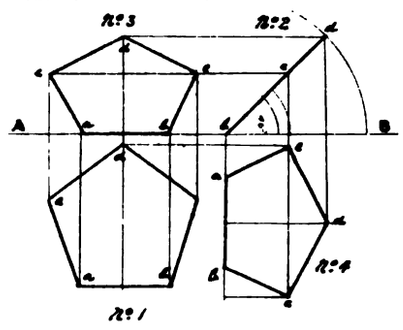
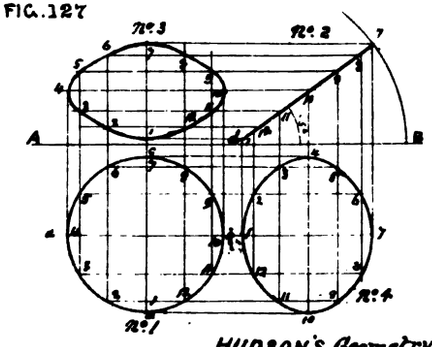


FIG. 127



ground line, make an edge view of the figure No. 2 inclined at 30°, *b a c* being made equal to the width across *b a c* (No. 1), measured on a perpendicular to the ground line. From *b a* and *c* (No. 2), draw lines parallel and perpendicular to A B for the heights in the elevation, No. 3, and the widths in the plan, No. 4.

Draw up lines from *a b* and *c* (No. 1), to cut those in *a b* and *c* (No. 3). Join *b a*, *b c*, and *a c*, which completes the elevation required.

Transfer the lengths of the triangle *a' b c'* measured on A B, to the perpendicular *b c'* (No. 4). Draw lines from these points parallel to A B, to cut the other parallels in *a b* and *c*. Join these points *a b*, *b c*, *c a*, which gives the plan required.

FIG. 126.

To draw the plan and elevation of a pentagon, when its face is inclined at any angle, as 40°.

Draw the common plan of the pentagon in any position, No. 1; and proceed with this figure as in the last case. These two examples will be found sufficient to show the method for determining any other figure with similar conditions.

FIG. 127.

To draw the plan and elevation of a circle, with its face inclined at any angle, as 35°.

Draw a circle, No. 1, and divide its circumference into any convenient number of equal parts, as 12. Mark these points 1 2 3, etc. Upon the ground line set off an angle of 35°, and upon it mark the parallel widths of the circle at the several points across *d c*, transferred to No. 2. From each of these points, draw lines parallel and perpendicular to the ground line, for the plan and elevation. If the divisions of the circle are equal and even in number, lines may be drawn from them in No. 1, parallel and perpendicular to the ground line, to cut the others in the plan and elevation, No. 3 and No. 4. The plan and elevation being completed by drawing the ellipses or curved lines passing through the intersections found. But if the divisions be irregular, the widths across *a d* (No. 1), must be transferred and marked across 4, 10 (No. 4), as in previous figures.

FIG. 128.

To draw the plan and elevation of a square or other figure when one of its sides is inclined at 30°, *and another at* 35°.

Proceed as in Fig. 120; that is, upon the ground line set off two lines, the length of the sides of the square, and inclined respectively at 35° and 30°, as *a c* (No. 1), and *a b* (No. 2). Draw a line *a c* (No. 2), at right angles to *a b*. From *c* No. 1, draw a line parallel to A B, to cut *a c* in *c'*, giving the height and length of this side in the elevation. Draw the other two sides of the square from *b* and *c*, meeting in *d*, parallel to the first.

For the plan, drop perpendiculars to the ground line from all the points (*a b c d*) in the elevation. Draw *a b* (No. 3), parallel to A B, and under *a b* (No. 2). From *c* (No. 1), drop a perpendicular to A B, and measure the parallel width from *a* to this line. From *a* (No. 3), with this distance as radius, draw an arc, cutting the parallel from *c'*, (No. 2), in *c*. Join *a c*, and draw the other sides of the square parallel to *a b* and *a c*. The transfer of the width *a c* (No. 3), is shown in fine lines.

FIG. 129.

To draw the plan and elevation of a square, when its plane is inclined at one angle, and one diagonal at another.

Upon the ground line set off an angle of, say, 45°, to represent the inclination of the plane *d c* (No. 1). Find the length of the diagonal of the square; and set it above, and inclined to the ground line (A B), at, say, 20°, as at *a b* (No. 2). Draw lines from *a* and *b*, parallel to A B, to cut the plane (No. 1), as at *a* and *b*. From the point in A B, where the plane *d c* produced cuts it as a centre, say *e*, then with *e a* and *e b* as radii, draw arcs cutting a perpendicular line drawn at *e*. At the points where these cut the perpendicular, draw lines parallel to A B, and between them place the diagonal again (*a' b'*, No. 2). Complete the square, thus: from *a'* and *b'* as centres, with the side of the square as radius, describe arcs cutting each other at *c* and *d*. Join these points *a' c*, *a' d*, *b' c*, and *b' d*. From *a' c b' d* (No. 2), draw lines parallel and perpendicular to A B. Transfer the two corners of the square *c* and *d* (No. 2), into the plane, No. 1, which is shown by the arcs. This is in effect simply turning the plane *d c*, or diagonal *a b*, into a vertical position, to obtain the front view and the other corners of the square, which are returned into the plane.

Draw lines from *d*, *a*, *b*, and *c* (No. 1), parallel and perpendicular to A B, giving the heights for the elevation and the width for the plan. The parallel widths across *a b*, are the same in No. 2, No. 3, and No. 4. Therefore transfer these widths (measured along A B) and draw perpendiculars extending into No. 3 and No. 4, to cut the other parallels. Join the points, *a c*, *c b*, *b d*, and *d a*, No. 3 and No. 4. Draw the diagonal *a b* in each view, which completes the view and elevation required.

NOTE.—If the conditions were reversed, the plan and elevation (No. 3 and No. 4) with the diagonal of the square being given in this position, to find the inclination of the plane and diagonal; the intersection of the two sets of parallels of width and length will determine the plane *d c* (No. 1). The inclination of the diagonal is easily found by placing it between the parallels *a a* and *b b* (No. 1 and No. 2).

FIG. 130.

To draw the plan and elevation of two planes which meet at right angles when equally inclined, and their line of intersection inclined at any angle, as 20°.

Determine the size (or a portion) of the planes, and draw them equally inclined at 45° (*a b*, *a c*, No. 1). Draw a line across *b c*, parallel to A B, and a vertical line *a d*. This view (No. 1) serves only to give the measurements for the others. Set off a line, *a c* (No. 2), inclined to the ground line at 20°. At *a* and *c*, draw perpendiculars to *a c*, each equal to *a d* (No. 1). Draw *b f*, parallel to *a c* (No. 2). From each of the points *a*, *b*, *c*, *f*, draw lines parallel and perpendicular to A B for the plan and elevation. Set off the widths across *c*, *d*, *b*, No. 1, as perpendiculars in No. 3, and parallels in No. 4 to the ground line, cutting the other parallels in the several points,

a b c e f g. Join these points with thick lines, and the plan and elevation required will be complete.

FIG. 131.

The plan and elevation of a square of one inch side given, to find the inclination of its sides, and also that of its plane.

This figure is the reverse of Fig. 129.

The plan of the square is placed with one side parallel to the ground line, to show more clearly the origin of the points, &c. *a b d c* No. 1 is the given elevation, and *a b d c* No. 2 the plan.

If the sides *a b c' d* No. 1 and No. 2 are not parallel to the vertical plane, another elevation of them in that position should be obtained, so as to find the point in which the sides cut the horizontal plane.

The square resting on one corner, and the two sides *a b* and *c' d* (No. 1) being parallel to the vertical plane (which will be seen by the plan or by measurement of the lines). Then *a b* (No. 1) is the true length of the line and angle of that side (30°). The angle of the other side, *a c'* is found by placing the full length of the line between the same parallels of height as *a c* (35°). Produce one side, as *d b*, to cut the ground line in *e*. Extend the same side, *d b*, in the plan (No. 2). From *e* (No. 1), drop a perpendicular to A B, cutting *d b* in *e* (No. 2). Draw a line, *e a* (No. 2), and produce it to *f*, then this is the line in which the plane, if extended, would cut the horizontal plane. An elevation on a plane perpendicular to this line (*e f*) will give the edge view and angle of the face of the square. At any point, as *a* (No. 2), draw a line, *a g*, perpendicular to *e f*. From each of the corners of the figure *a b c d*, draw parallels to *e f*, cutting the perpendicular in *g h i*. Mark off the distances *g h i a* on the ground line at No 3, and draw perpendiculars. Draw parallels to A B from the points *a b c' d* (No. 1), cutting the others in one straight line *a b c d* (No. 3). This will be the true angle of the plane of the figure.

The points *a d* (No. 3) alone will determine the plane without the intermediate points *b c*. No. 4 is a front view of the square when turned up.

FIG. 132.

To draw the plan and elevation of a cube of one inch edge, when two adjacent sides are equally inclined, and its base inclined at any angle, as 30°.

Draw the plan of one face of the cube, as No. 1, the edges forming equal angles with the ground line (A B). On A B (No. 2), set off a line at 30° for the base, or set up a line at 60° for the edge of the cube, one being at right angles to the other. On *a c* (No. 2) mark off a distance equal to the diagonal of one face. Draw the three lines *a e*, *d h*, and *c g*, each equal to one edge, and perpendicular to *a c*. Draw a line, *e g*, parallel to *a c*.

Having obtained two views (No. 1 and No. 2) of the solid, the plan and elevation may always be determined by the intersection of parallels drawn from every point in each view. The elevation, No. 2, is found by the intersection of parallels of height from No. 2, with the parallels of width in No. 1. The plan No. 4 is found in the same way, by the parallels of length in No. 2 with those of breadth in No. 1.

The same letters being affixed to the same points in each view, the points may be easily determined and joined in thick lines, giving the outline of the figure or solid required.

FIG. 133.

To draw the plan and elevation of a cube, when the inclinations of its base and one edge, or that of two edges, are given.

Let the given inclination of its base be 55°, and one edge of that base 45°. Then four of the edges will be inclined at 35°, because they are perpendicular to the base; therefore, when the angles of two edges are given, the inclination of the base is found.

On A B (No. 1) set off a line for the base *a b* at 55°, and a line in the opposite direction, *a c*, at right angles for one edge. Draw a line, *a d'*, at 45°, to determine the height of the point *d'*.

From *d'* draw a parallel to A B, cutting the base *a b* in *d*. At *d* erect a perpendicular to *a b*, and also one at *a*, each equal to one edge. Join *c f* and produce this line. Then *a d f c* will be one face of the cube, and this fixes the position of the solid. Find an end view of the base, so as, to determine the other lines thus :—On *a* (No. 1) as a centre, with *a d* as radius, mark off this distance upon a vertical line, as at *d'*. Project a line from *d'* parallel to A B, and place one edge between these parallels, as *a d* (No. 2). Construct a square on *a d*, and from the other corners *b* and *h* draw parallels to cut *a b'* (No. 1). Transfer the points on *a b'* to the base *a h b* (No. 1). Draw the lines *h g* and *b e*, completing this view of the solid. Then, from views No. 1 and No. 2, the plan and a second elevation may be found, as in other figures. The plan No. 3 is found by the intersection of the vertical parallels from No. 1 with those of No. 2, transferred from *i j*, *a k*, on A B, and marked off upon a line at right angles to A B.

The elevation (No. 4) is found by the intersection of the horizontal parallels from No. 1, produced with the vertical parallels transferred from No. 2. The widths in No. 2, No. 3, and No. 4 are the same, which will be seen by the fine lines.

FIG. 134.

To draw the plan and elevation of a cube, when the inclination of two of its faces are given.

Let the given angles of two faces be 35° and 70°. On the ground line at *a* (No. 1) set off a line at 70°, as *a h*, and another *a b* (No. 2), at 35°. At *a* (No. 2) draw a line *a c*, perpendicular to *a b*, and equal in length to the edge of the proposed cube. Project the point *c* (No. 2) into the plane *a h* (No. 1), as at *c*. Turn up the point *c* to *d*, on *a* as a centre, and obtain a front view of the face to which this edge belongs in the following manner. Draw a line from *d* (No. 1) parallel to A B. Place the edge *a e* between these parallels, and upon it construct the square *a e g f*. Turn the other corners, *f* and *g*, into the plane *a h*, as shown.

From the point *f* in *a h*, draw a line to cut the base *a b* (No. 2) in *d*. Draw *d e* parallel to *a c*, and *c e* produced parallel to *a b*. Then *a c e d* is one face of the cube, inclined at 70°; and from these the rest of the cube is easily determined, as in the previous Fig. 133, No. 2.

FIG. 128.

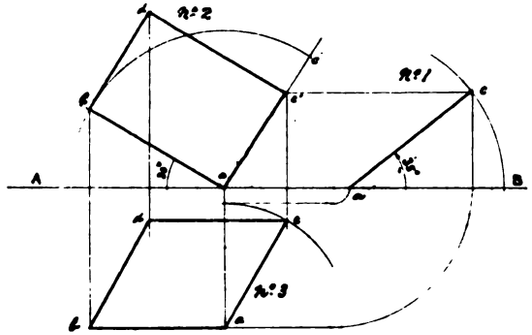


FIG. 129.

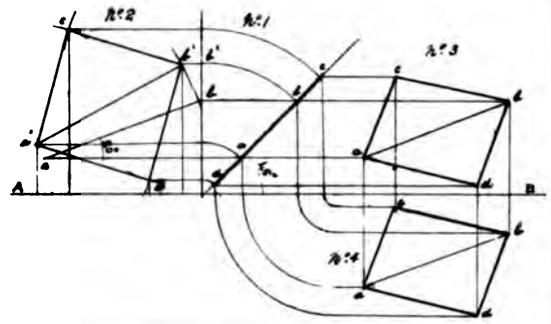


FIG. 130

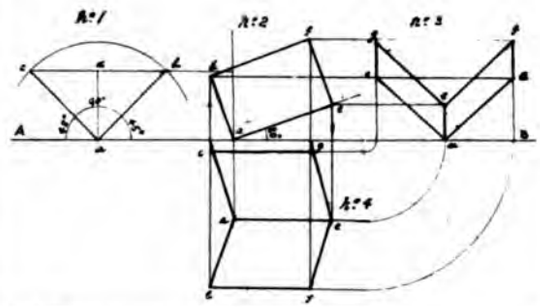


FIG. 131

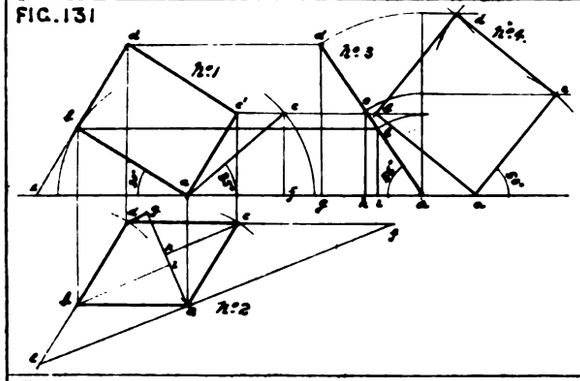


FIG. 132.

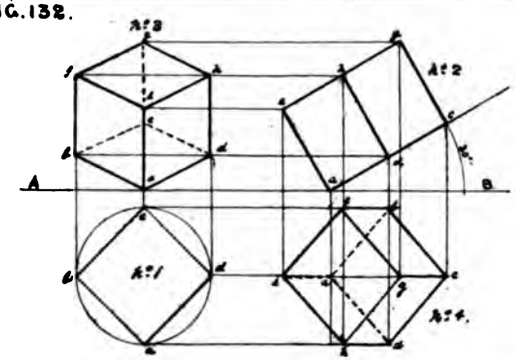


FIG. 133

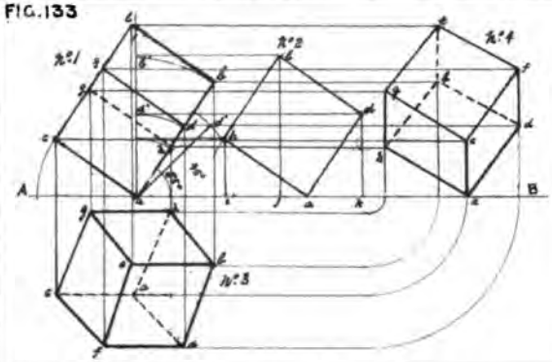


FIG. 134

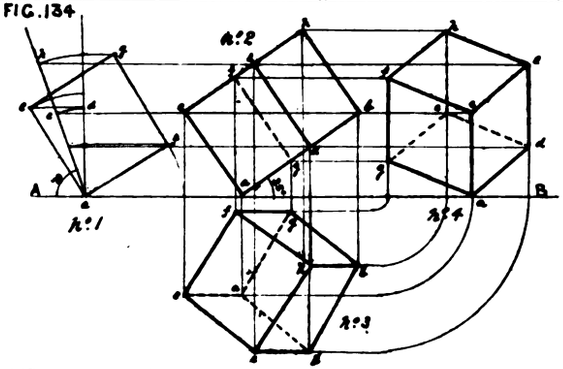


FIG. 135

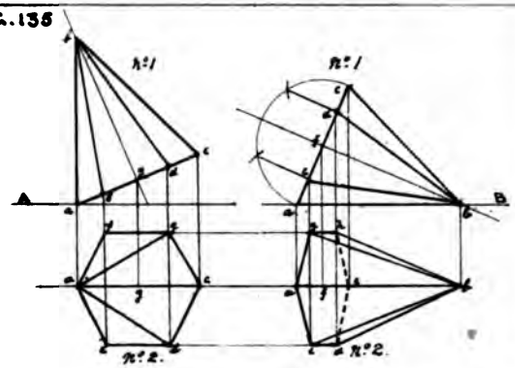
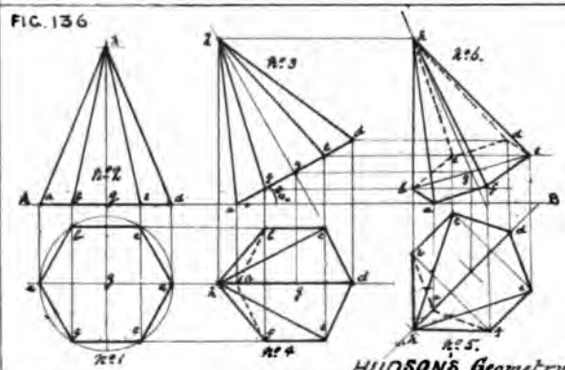


FIG. 136



HUDSONS. Geometry.

The following is another method for finding the other points in this view (No. 2), but not shown in the figure for want of space. Produce *a c* and *d e* (No. 2), using *a c* as a ground line, and between these parallels place one edge; upon it construct a square. Draw lines parallel to *a c*, from the other corners, to cut *a b*, in *g* and *b*; part of these lines will form the other edges of the cube required.

The plan and elevation, No. 3. and No. 4, are found in the usual manner, as previously described and shown in fine lines.

FIG. 135.

To draw the plan of a given pyramid in any fixed position.

1st. Let No. 1 be any given pyramid, with one long edge vertical, with the two adjacent faces equally inclined, and its base a regular hexagon.

2nd. Let No. 1 be the same pyramid, with one long edge horizontal, and the adjacent faces equally inclined.

From all the points in these views (No. 1), draw perpendiculars to A B, and produced for the plans. On the base *a c* (No. 1) as a diameter, draw a semicircle, and divide it into half the number of parts that there are sides for the base (as 3). Measure the distances from the line *a c*, to the points in the semicircle, and transfer them to the plan, setting them off on each side of the centre line *a b* (No. 2). Produce these lines to cut the former in the points *a b c*, etc. Join these points to complete the plans as shown. If another view is desired, proceed as in previous figures.

FIG. 136.

To draw the plan and elevation of a pyramid, when either its base or its axis are inclined at any given angle; also to draw a second elevation upon a plane making any given angle with the first.

Draw the base, No. 1. Project lines from all the points in No. 1, perpendicular to, and cutting the ground line in, *a f c d*. Add the centre line, *g g*, and mark the height *g h* (No. 2). Join the corners of the base with *h*.

On the ground line at *a* (No. 3) set off a line to represent the base of the pyramid inclined at any angle, as 30°. Copy the points, *a f g e d* (No. 2) on to this line. Draw a perpendicular at *g*, for its axis. Mark the height and join all the points, completing the view No. 3.

From every point in No. 1 and No. 3 draw lines parallel and perpendicular to the ground line, so as to cut each other in the points *a b c d e f h* (No. 4). These form the plan of the solid in this position.

To find an oblique, or second elevation, of the solid in this position, upon a plane making any given angle with that of the first elevation.

Draw the plan No. 5, which is the same as in No. 4, but having its central axis *d h*, inclined to the ground line (A B) at the required angle. From the points in No. 3 and No. 5, draw the parallels to cut each other in No. 6. Join the points with thick lines; then this view will be the second elevation sought. The same points in each view should be marked with the same letters, to prevent confusion in following and determining the several forms.

FIG. 137.

To draw the plan and elevation of an oblique pyramid, with a square base; the angles of three of its faces being given, and its base inclined.

Let the base be a square of ·75 inch side, the angles of the three faces 75°, 60°, and 85°. The inclination of the base 40°.

On A B set up a line, *a c* (No. 1), at 75°. Mark the length of the base on A B; and at the ends of this line *a b* (No. 2), draw others inclined at 60° and 85°, to represent two opposite faces, cutting each other in *c*, and giving the height of the solid. Draw the plan No. 3 as follows: mark down the size of the base. Drop a perpendicular from *c* (No. 1), and measure the distance from this line to *a*, along A B, which will be the distance of the apex *c* (No. 3), from the line *a b*. Join the corners of the base *a b d e* with the point *c*. Repeat the elevation No. 2 at No. 4, setting the base inclined at 40°. Draw the parallels from No. 3 and No. 4, to cut each other in *a b d e c*, No. 5. Join these points by thick lines, which will complete the plan. The inclination of the fourth face may be found in the following manner. The width from *c* (No. 3) to the line *d e*, and the height *b c* (No. 2) being already determined; a line placed between these parallels will give the true angle.

The inclination of two adjacent faces only, and the height, are sufficient to draw the solid.

FIG. 138.

To draw the plan and elevation of a given cone, when its base or axis is inclined.

Draw a circle for the base of the cone No. 1, and divide it into any number of equal parts, 1 2 3, etc.

NOTE.—The radii are simply shown as a means of dividing it.

From the points in No. 1 draw parallels and perpendiculars to A B, for No. 3 and No. 4. Set off a line 1 *c* (No. 2,) at an angle equal to the required inclination of the base. Transfer the widths of the parallels measured across 1 *a* 7 (No. 1,) to the line 1 *b* 7. (No. 2.) Draw a perpendicular at *b*, for the axis, and on it mark the height of the cone, *b a*. Join *a* 1 and *a* 7. Draw parallels A B from all these points in No. 2, to cut those from No. 1, for the elevation No. 3. Through the several points, where these lines cut, trace the curved line of the base, which is an ellipse. Draw two lines from *a*, (tangents to the ellipse,) for the sides. The plan No. 4 is found in the same way, by the intersections of the other parallels from No. 1 and No. 2. (Fig. 127.)

FIG. 139.

To draw the plan and elevation of any compound solid, as a square plinth, and pyramid, in any given position.

Let the conditions of position, etc., be as follows:—Its base inclined at 30°. One side of the plinth inclined to the plane of the elevation, at 20°. The common plan (No. 1,) and the heights of the plinth and pyramid, as in No. 2.

Turn the plan No. 1 into the given position, with one side forming an angle of 20° with the ground

line. Draw a datum line Y Z, perpendicular to A B, and draw parallels from all the points in No. 1, to meet this line. On A B (No. 2) set off a line at 30° for the base, and upon it copy the widths from the line Y Z. Mark the given height of the plinth and pyramid. Join all the points necessary to complete the view, No. 2.

From the several points in No. 1 and No. 2, draw lines parallel and perpendicular to A B, to cut each other in No. 3 and No. 4. Join all the points as shown, which will give the plan and elevation, as in previous figures.

FIG. 140.

To draw the plan and elevation of a given sphere or globe, having upon it lines of latitude and longitude, when its axis is inclined.

Let the given inclination of the axis be 60°, and No. 1 the given plan of the sphere with the longitudinal lines or radii.

Draw the axis, *b a*, (No. 2,) inclined at 60°, and the circle *a b* for the sphere. Divide the circumference of the circle, and draw the required latitudes, 9, 13, 1, 5, and 9, 13. Find the diameter of the circle 9, 13, (No. 2,) and inscribe it in No. 1.

Figure all the points of intersection in No. 1 and No. 2 to correspond. Project parallels from the several points in these two views to cut in No. 3 and No. 4. Trace out the curves (or portions of ellipses) to complete the required plan and elevation as shown.

FIG. 141.

To draw the plan and elevation of any given irregular solid, as a ratchet-wheel, when its face or its axis is inclined.

This figure combines all the principles laid down in the preceding examples given in Solid Geometry; description is therefore unnecessary.

FIG. 142.

To draw the plan and elevation of a pyramid of four equal spheres, all in contact.

Draw the plan (No. 1), of the spheres, three lying together, with another resting upon them, and in such a position that the line or plane passing through the centre of the upper, and one of the lower spheres shall be parallel to A B. Then the elevation of these spheres *a* and *d* (No. 2), are readily found, by projecting the lines of No. 1 into No. 2; describing the circles in contact with their centres in these lines, as *a d b*. The exact point where the spheres touch at *e* is determined by the line joining their centres. No. 3 is a second elevation, taken at right angles to the first, which is found by transferring the widths and heights from No. 1 and No. 2, as shown by the fine lines.

FIG. 143.

To draw the plan and elevation of a pyramid of five equal spheres.

Draw the plan of the spheres, four lying together, with another resting upon them, and in the same position as in the previous figure. The elevation No. 2 is determined by projecting the lines from the plan, and drawing the three circles, *a e c* in contact.

The other elevation (No, 3) is readily found.

FIG. 144.

To draw the plan and elevation of a given cluster of spheres of various sizes.

On A B draw circles in contact, to represent the spheres, as *a b c* (No. 1). Draw perpendiculars to A B, through the centres. Draw the centre sphere *a*, in the plan No. 2. Take the distance between the parallels as radius, and from *a* (No. 2), as a centre, draw arcs, *b* and *c*. Determine the position of the spheres, *b* and *c*, in the plan No. 2. Square up lines from *b* and *c* into the elevation, and upon these lines draw the spheres *b* and *c*; this completes the elevation. The profile of the spheres gives the distance between the centres for the plan; and by joining the centres, the exact point of contact may be found. The plan and elevation of any number of spheres rolled together may be determined in this way.

FIG. 145.

To draw the plan and elevation of a regular tetrahedron, when one face is given.

This solid has four equal faces, each equilateral triangles.

Let *b c* and *d* (No. 1) be the given face. Find the centre *a* of the triangle, and join it with the corners *b c* and *d*. Draw the ground line A B, parallel with one of these lines, as *a b* (No. 1). From the points in the plan draw up the parallels for the elevation. At *b* (No. 2), as a centre, with the length of one edge *b c* (No. 1), for radius, cut the centre line in *a*, giving the height of the solid. Join *a d*. The elevation in this case coincides with the vertical section through *a b* produced (No. 1).

FIG. 146.

To draw the plan and elevation of a cube or hexahedron, one face being given.

This solid is composed of six equal squares. Draw a square for the given face *b c d e*, (No. 1,) which is the plan. The elevation No. 2 is projected from the plan. The height is equal to one edge or face.

FIG. 147.

To draw the plan and elevation of a regular octahedron, one face being given.

The octahedron is a solid composed of eight equal equilateral triangles.

NOTE.—In this, as well as in the other solids, the planes of the opposite faces are parallel. The polygons of the faces have their corners alternating in direction. The plan of the octahedron is consequently a regular hexagon.

The elevation is, in all cases, found by drawing a vertical section, which gives the height of the solid, and also the distance between the parallel faces.

Draw an equilateral triangle, *e f g*, (No. 1,) or the given face. Draw the opposite and parallel face *b c d*, (No. 1,) by circumscribing the triangle with a circle; divide it into six equal parts, and join each alternate point. Complete the plan by joining the points *b f c g d*. (No. 1.)

Draw the ground line A B, parallel to one diagonal, as *g b*. From the several points in the plan, draw up parallels for the elevation. At *g* and *e*, (No. 2,) as centres, with the length of one face (measured from

FIG. 137

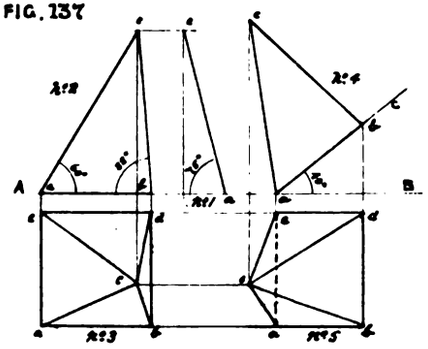


FIG. 138

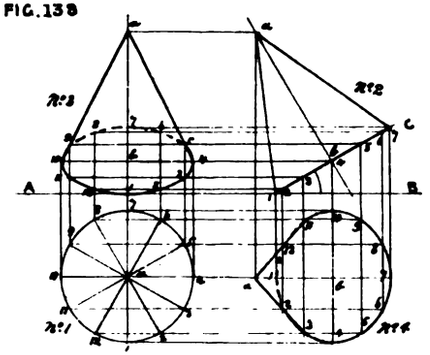


FIG. 139

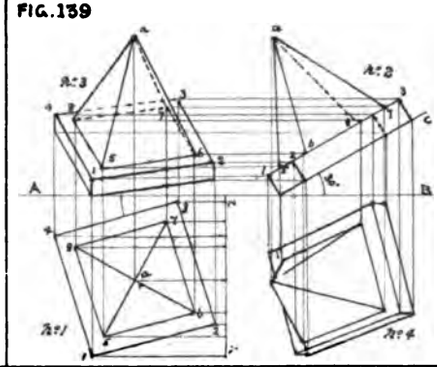


FIG. 140

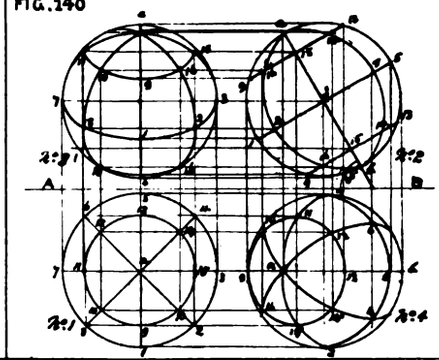


FIG. 141

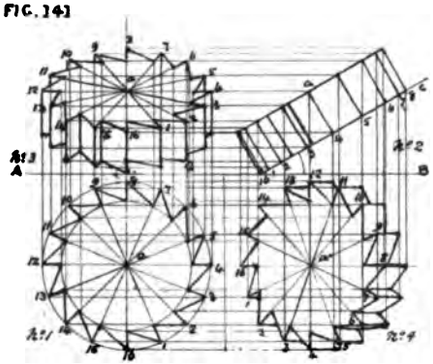


FIG. 142

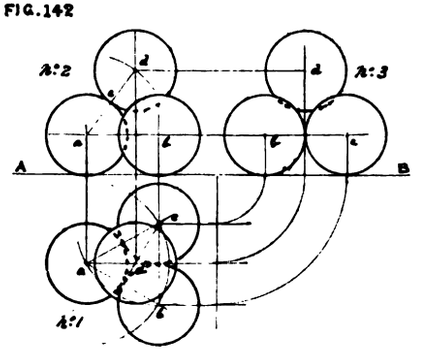


FIG. 143

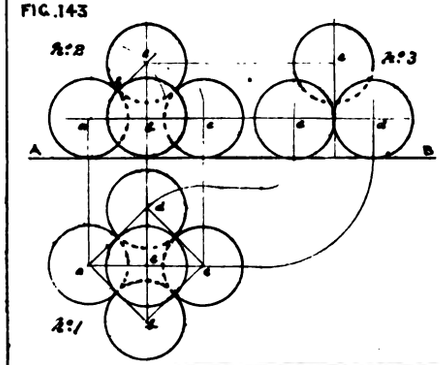


FIG. 144

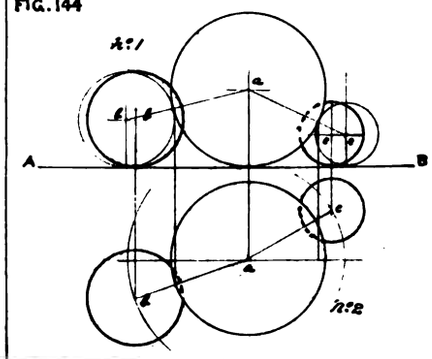


FIG. 145

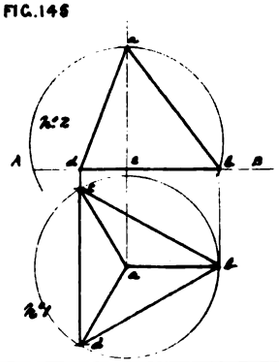


FIG. 146

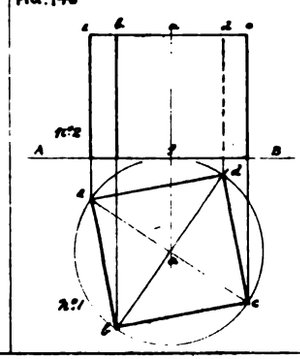


FIG. 147

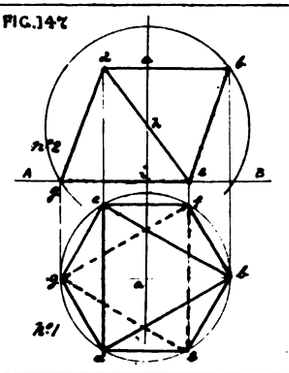


FIG. 148

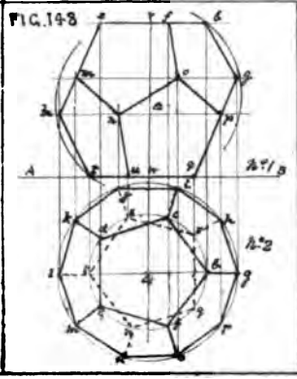


FIG. 149

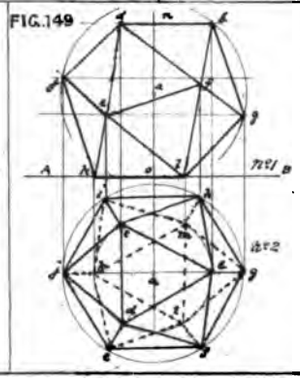
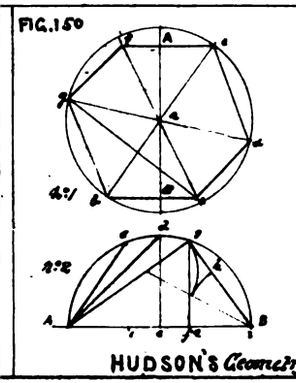


FIG. 150



point to side) as radius, cut the parallels in *d* and *b*, giving the height of the solid. Join *g d*, *e b*, *d b*, *g e*, and *d e*. From this data other elevations may be drawn.

FIG. 148.

To draw the plan and elevation of a regular dodecahedron, one face being given.

This solid is composed of twelve equal faces, each a regular pentagon. Let *b c d e f*, (No. 2,) be the plan of the given face. Draw another face, *r s t u q*, with its corners opposite to the sides of the other. Then all these points will lie in one circle, described from *a*, as a centre.

The other corners, *g h i - - - - - - p*, all lie in the circumference of a circle described from the centre *a*, with a radius, *a g*, equal to one edge, *b c*, increased by its minor (or smaller) segments, when medially divided. (See Fig 50.)

Draw radial lines from the centre *a*, through the corners of the first two polygons, cutting the outer circle in the ten points *g* to *p*. Join all these points round the outer circle, completing the plan.

For the elevation, No. 1, draw the ground line A B, parallel to one diagonal, as *l g* (No. 2). Project parallel lines from all the points in the plan, into the elevation. Then *t u q*, (No. 1,) is the base. Take *b c*, the edge of one face, as radius, and from *t*, (No. 1,) as a centre, cut the outside parallel in *l*. Next take the length of one face (measured from point to opposite side) as radius, and from *l*, as a centre, cut the parallel from *e d*, (No. 2,) in *e*, (No. 1,) giving the height of the figure. Draw *e b*, the upper face, parallel to the base, A B; *g q*, equal to *l e*; and *g b*, equal to *t l*. This outline will be a section of the solid through *l g* (No. 2). Draw lines through *g* and *l*, parallel to A B. Mark the points *m n o p*, where the parallels cut. Join the points *m n*, *n o*, *o p*, *o f*, and *n u*, completing the elevation.

FIG. 149.

To draw the plan and elevation of a regular icosahedron, one face being given.

The icosahedron is a regular solid, having twenty equal equilateral triangles for its faces. The construction of this figure is similar to the former one; each edge, as *b c*, of one face, and each alternating point, as *m*, of the opposite and parallel face, being three angles of a hexagon. Hence, in the plan, the horizontal diagonal *i g*, is made equal to the parallel edge, *b c*, increased by its major (or longer) segment, when divided medially. The section, or outline of the elevation, is obtained as in the previous figure. (See Fig. 150.)

FIG. 150.

The sphere being given, to find the edge of any polyhedron that can be inscribed in it.

Let A B, (No. 1,) be the given sphere. On A B, (No. 2,) draw a semicircle, equal in diameter to the sphere. Make B $f = \frac{1}{3}$ of A B; (No. 2). Draw *f g*, and *c d*, at right angles to A B. From A, draw a line (which is not shown) perpendicular to A B; and equal to it, draw a line from *c*, to the extremity of this perpendicular, cutting the circle in *e*. Join A *e*, A *d*, A *g*, and B *g*. Then the chord A *g* is the edge of the tetrahedron, B *g*, that of the cube, A *d* that of the octahedron. Divide B *g* medially in *h*, and the major segment B *h*, is the edge of the dodecahedron, and the chord A *e*, that of the icosahedron.

The following relations between the several lines and planes of these solids, are worth remarking:— When the diameter of the great circle, or circumscribing sphere is the same for each solid, the edge of the tetrahedron is equal to the diagonal of a face of the cube. The edges of the octahedron are those of three squares, the plan of each of which is at right angles to those of the other two, and each two of which have a diagonal in common. The diagonal of the face of the dodecahedron, is equal to the edge of the cube, and the distance between the parallel faces of the dodecahedron and icosahedron, is the same for both solids. The lines in No 1, refer to the section, or outline of the dodecahedron and icosahedron, their sections being alike when they are inscribed in the same sphere.

PENETRATIONS AND INTERSECTIONS OF SOLIDS.

PLATE 12.

FIG. 151.

A cube being given, to determine the inscribed and circumscribed spheres in plan and elevation.

The inscribed sphere touches the centre of each face of the cube. The circumscribed sphere touches all the corners of the cube; therefore the centre is common to both.

Draw the face of the given cube, *a b c d*, (No. 1,) in the position shown. Find the elevation of the cube, *a b c, e f g*, (No. 2,) and draw the diagonal *a g*, which is the diameter of the circumscribing sphere. Therefore, a circle drawn from *h*, as a centre, and passing through the corners *a e g c*, will be one sphere. The diameter of the other sphere is equal to one edge of the cube *b f*. Draw these circles in the plan and elevation.

FIG. 152.

A tetrahedron being given to determine the inscribed and circumscribed spheres in plan and elevation.

Draw the plan of the given tetrahedron *a b c d*, (No. 1,) in the position shown. Find the elevation *a b d*, (No. 2.) Bisect the edge, *a d*, by a perpendicular, cutting the centre line *d d*, in *e*. From *e*, as a common centre, with radii *e f* and *e a*, draw two circles, one touching *a b*, and *b d*, the other touching the corners *a* and *d*. Draw these circles in the plan, (No. 1,) which represent the spheres.

FIG. 153.

To draw the plan and elevation of any two cylinders of given diameters, one pierced by the other through its centre, and determine their lines of intersection.

Draw the elevation, No. 1, of the given cylinders,

in any position, at pleasure. Draw the centre lines C D, and E F. At one end of each cylinder, as at C and E, draw a semicircle equal to its diameter, and divide the circumference into any convenient number of equal parts (as six), in each case. From all these points, draw parallels to the centre lines. These lines will serve both for projecting the plan, and determining the lines of intersection.

Draw a centre line for the plan, parallel to A B; at one end repeat the semicircle at E, and at the other, that at C. From the points in the smaller semicircle, draw lines to cut the larger in 1 2 3 4 5 6 7. Draw the chords 1, 7; 2, 6; 3, 5; perpendicular to the centre line. Transfer the widths between these chords, to the elevation, as shown on C D, and trace the curved line 0 1 2 3 4 5 6, through the intersection of these lines, with those drawn from D. Repeat this curved line on the other side of the large cylinder. If the cylinders are equal in diameter, and cut each other centrally, their intersections will form straight lines, instead of curves. The plan, (No. 2,) is obtained in the usual manner. Draw down the lines from the elevation, and draw the parallels of widths to cut each other in points, forming ellipses. (See Fig. 127.) The semicircles and lines previously drawn, serve also for the plan, if divided alike.

FIG. 154.

To draw the plan and elevation of any frustum of a cone pierced by a square prism.

Let *a b c d*, No. 1, be a given cone, or frustum, and *e f g h*, the square prism. Draw the plan of the cone and prism, (No. 2,) also two or more circles to represent sections[1] through 1, 4, and 3, 6 (No. 1). From the points where these circles cut the prism, draw lines into the elevation, cutting *f g* in *j k*, and *e h* in *i l*. Join *i j*, and *k l*, the lines of intersection of the two solids.

FIG. 155.

To draw the plan and elevation of any cone, or frustum, pierced by a cylinder.

Let *a b c d*, (No. 1,) be a given cone, and *e f g h*, the penetrating cylinder. Draw their outline in the plan (No. 2). At the junction of the centre lines of the cylinder and cone, draw a semicircle equal to the cylinder (No. 1). Find the apex, or point of the cone, and from it draw a line, *t s*, to touch this semicircle. Then the point *s*, will give the limit of the curves. To find the curved lines, formed by the intersection of the surfaces, draw lines to represent horizontal sections, as *i j*, and *m r*. Draw circles in the plan equal in diameter to the cone, measured on the lines, *f g*, *i j*, *o q*, *m r*, and *e h*. On the line *m r*, measure the distance from the centre line of the figure, to the point where this plane cuts the semicircle, and mark off this width in No. 2, on each side of the centre line, *c d*. Draw lines *s t*, and *u v*, through these points, parallel to *c d*. Where the lines *s t* and *u v* cut the circle (equal in diameter to *m r*, No. 1), will be four more points in the curves; and these points projected into the elevation, will cut *m r* in two points, *s* and *t*, of the curves. In the same way find the points *i j* *k l* (No. 2) and *k l* (No. 1) belonging to the second conceived section through *i j* (No. 1). Draw the curved lines through the points found in the plan and elevation. In this case, eight points in each of the curves were determined; but, by the addition of other horizontal sections, more points may be found.

[1] These sections may be taken at random.

FIG. 156.

To draw the plan and elevation of any sphere pierced by a given cylinder.

Let No. 2 be the plan of the solids; the centres of the cylinder and sphere not being common to each other. Draw the outline of the elevation No. 1, by projecting the lines from the plan. Join the centres *e c*, (No. 2,) and produce this line to cut the circle *p*. Draw lines, *f g*, and *a b*, (No. 2,) through the centres of the two solids, and parallel to A B; draw also *l m*, to touch the cylinder at *d*, for the limits of the curves and any other lines, as *j k*; then all these lines will represent vertical sections, which are indicated in the elevation (No. 1), by circles, equal in diameter to those lines. From the points *a n d p o* and *b* (No. 2), draw lines to cut the circles in *c l k j d*; and *e o m n f* (No. 1). Trace curved lines through these points, forming the intersection of the solids. The front portions only of the curves are shown, the others are found in the same way. When the centre of the cylinder passes through the centre of the sphere, their intersection is shown by straight lines. This occurs also with the cone and sphere.

FIG. 157.

To draw the plan and elevation of any sphere pierced by a given cone.

Let *b c* and *n g* (No. 1) be the diameters of the sphere and cone, *a* and *c* being their centres. Join the centres and produce the line to cut the circle at *f*. Draw the outline of the elevation of the sphere and cone No. 2. Find a vertical section through *a f*, (No. 1,) as follows: On *a*, as a centre, with *a e* and *a f* as radii, draw arcs to cut *b c* produced. Project these points into the elevation, and join *j k*, cutting the sphere in *o* and *m*. Then the lines *o h* and *m m*, give the limits of the curves, in one direction; for the others, draw a line *g h*, (No. 1,) through the centre of the cone, *e*. The elevation of this section will be a circle, cutting the sides of the cone, giving the extremities of the curves. Draw lines *g h*, and *e f*, No. 2, and conceive sections through these points. Show their plans by circles, *i m*, *j l*, *a d* and *o*. From the points where these circles cut one another at *n p*, (No. 1,) and other points not marked, project lines into the elevation to cut the same planes. Through these points in No. 2, draw the curved lines of the intersection of the solids.

FIG. 158.

To draw the plan and elevation of any given sphere pierced by a hexagonal prism.

Let *a c*, (No. 1,) be the given sphere, and *e f g h i j*, the hexagonal prism in the plan. Project the elevation *a b c d*, (No. 2,) *e* being the centre of the sphere. Draw the lines *k l*, and *m n* (No. 1); one touching the face *i j*, and the other passing through the middle of the two faces, *h i* and *e j*. From *e*, (No. 2,) as a centre, draw circles equal in diameter to *k l* and *m n* (No. 1).

FIG. 151

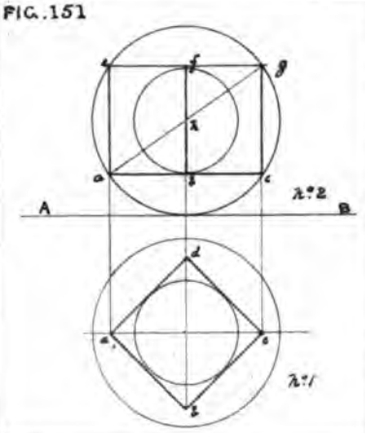


FIG. 152

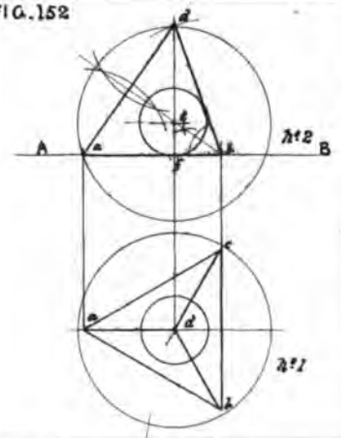


FIG. 153

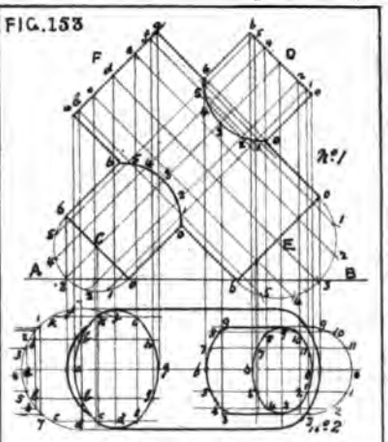


FIG. 154

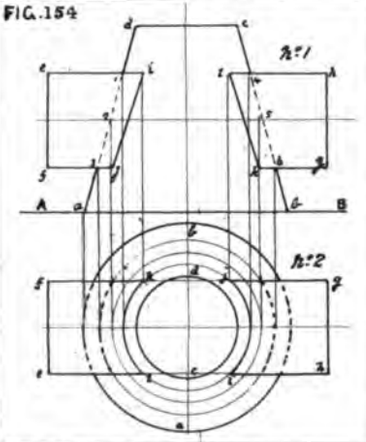


FIG. 155

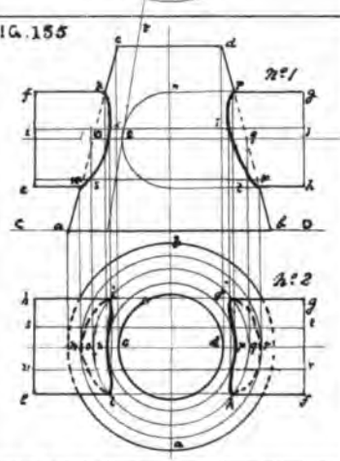


FIG. 156

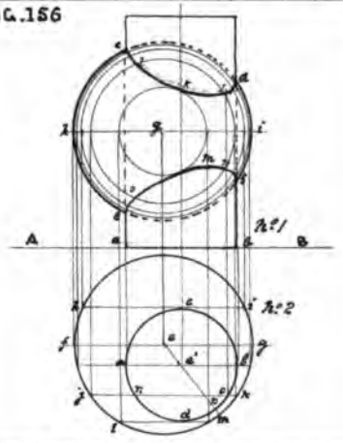


FIG. 157

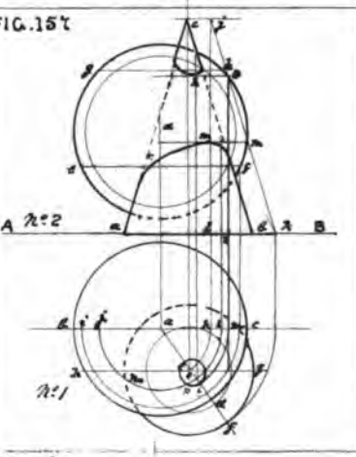


FIG. 158

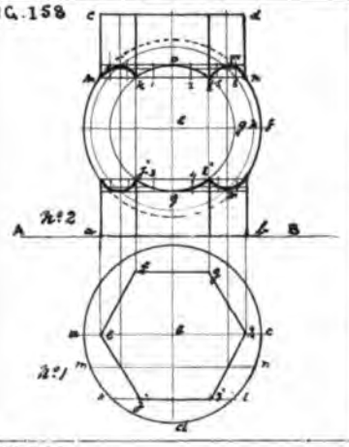


FIG. 159

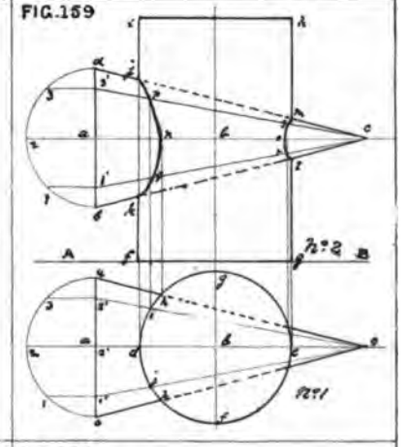


FIG. 160

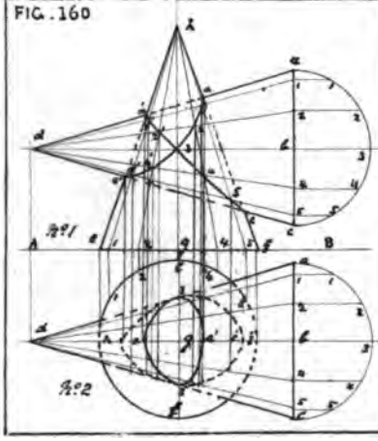


FIG. 161

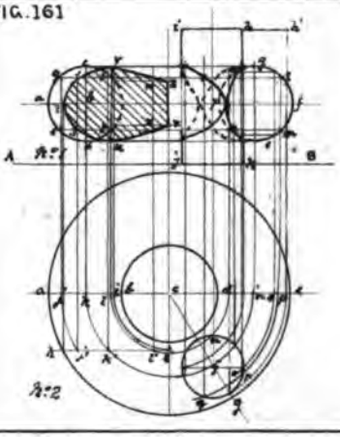
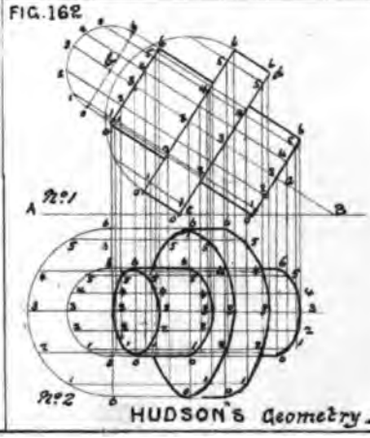


FIG. 162



Draw lines from the middle of each face in the plan, to cut the circles in *o p q r*, etc., (No. 2,) and giving the limits of the curves. Then the portions of the circle *j q i* and *k o l* will be the lines of intersection of the sphere, with one face of the prism. Determine any points in these arcs, and draw lines 1 2 and 3 4 through these points, parallel and perpendicular to A B. Produce the parallels each way. Draw similar lines 5 and 6 in the other faces to cut the former, and trace curved lines through these points.

FIG. 159.

To draw the plan and elevation of any cylinder pierced by a cone.

Let *d f e g*, (No. 1,) be the plan of the given cylinder, and *o* 4 *c* the cone, with its axis *a c* parallel to A B. Draw an elevation of the same, as *f g h i*, and *b c d*. (No. 2.) Draw the centre lines, and upon *o* 4 (No. 1) and *b d* (No. 2) as diameters, draw semicircles, and divide them alike, into any *even* number of parts, as at 1 2 3. From these points project lines parallel to *a b*, to cut the base of the cone, as at 1′ 2′ 3′ in No. 1 and No. 2. Draw lines from these points 1′ 2′ 3′, to the apex *c*, of the cone. Where the last drawn lines cut the circle in *h i d j k* (No. 1) draw up lines to cut those in No. 2 at *k q n p j*, and trace a curved line through these points. Repeat this process at the other side of the cylinder, giving the curve *l r o s m* (No. 2). The same method applies when the centre of the cone does not coincide with that of the cylinder.

FIG. 160.

To draw the plan and elevation of any two intersecting cones.

Let *a d c*, and *e f h*, (No. 1,) be the outline elevation of the cones. Draw their plan as *a c d* and *e i f h*. (No. 2.) On *a c*, the base of the cone *b d*, in No. 1 and No. 2, draw semicircles, and divide them alike into any *even* number of parts, as 1 2 3 4 5. From these points draw parallels to *b d*, to cut *a c*, and from thence draw lines to the point *d*, in both views. Divide half of the circle *h i* (No. 2) into the same number of parts, and draw lines from these points to cut A B in 1 2 4 5 at the base of the cone; also from these points to *h*, cutting the lines of the other cone at 1 2 3 4 5 and 1′ 2′ 3′ 4′ 5′, giving the curves of their natural intersection, in the elevation No. 1.

Drop perpendiculars to A B, from each of the points in the curves of No. 1, to cut the lines already drawn in the plan, in points through which the curved lines, *a* 3, *a′ c′*, etc., may be traced.

FIG. 161.

To draw the plan and elevation of a circular ring (or torus), penetrated by a cylinder; also to find the form of any section of the ring.

Let *a c g f e d* (No. 1) be the outline of the ring in elevation; and *a b c d e* (No. 2) the plan. Determine the size and position of the cylinder, *f*, in the plan, and project its elevation *h i j k* (No. 1). To find the lines of intersection, imagine a plane passing through the centre of the ring *a c*, and the axis of the cylinder, as *c g* (No. 2). Mark the points in which this plane cuts through the cylinder. Turn the plane round on *c*, as a centre, into the position *c e*; as shown by the arcs. Draw up a line from *p*, in *c e*, to cut the ring in *l m* (No. 1), giving the limits of the curves. Determine any points in the cylinder as *m r g q*, etc. (No. 2.), and turn these points round on *c*, as a centre to *d n o*. Draw up lines, as before, to cut the ring. From all the points *m o r g q* (No. 2), project perpendiculars to A B, to cut lines drawn parallel to A B, from the points in the ring *g l f m e*, etc., and giving points, through which draw the curves required. The form of a section of the ring, at any part, as *h i′* (No. 2), is found in the elevation in a similar manner to the foregoing; that is, by determining any points, as *j′ k′ l′* (No. 2), and finding their positions, 1 2 3 *u v w x y z* (No. 1), giving the outline of the section shown by shade lines in the plate.

FIG. 162.

To draw the plan and elevation of a cylinder encircled by a collar.

Let *a b* (No. 1) be the cylinder, having a collar *c d*, round the centre of it, the solid resting on two points *c o* (No. 1), or when its axis is inclined at a certain angle.

The elevation (No. 1) being easily determined, find a number of points, 1, 2 3, etc., on its edges by means of semicircles similarly divided, as in Fig. 153. Repeat the semicircles in the plan (No. 2), drawing lines from the points parallel to A B. Draw down lines from all the points in No. 1, to cut those in No. 2 in the several points 1 2 3, etc., and forming ellipses in the plan. Draw the sides of the solid, darkening the outline, as shown.

CONIC SECTIONS.

FIG. 163.

Let *a b c* (No. 1), be a *right cone*. The base *a b*, being perpendicular to the axis *c*. Then any section, as *d e*, parallel to the base, will form a circle. No. 2 is the plan of the cone so cut.

When the base of any cone is not perpendicular to the axis, it is called an *oblique cone*. The part, *a b d e*, is termed a frustum.

FIG. 164.

To draw the plan of a given frustum of a cone, cut at any angle; also to find the true form of the section.

Let *a b c* (No. 1) be the given cone cut by the plane *d e*, and leaving the frustum *a b d e*. Draw any number of lines parallel to *a b*, to represent horizontal sections, cutting *d e* at 1 2 3 4 5. Then commence the plan (No. 2), by drawing the base *a b*; add circles, each equal in radius to the sections *k j i l m* (No. 1), shown by the lines drawn from these points. From the points 1 2 3 4 5 (No. 1), draw lines to cut the circles in the plan, as at *d*, 1 2 3 4 5 *e*. Join these points by a curved line; and shade the part enclosed, to indicate the portion so cut by the plane *d e* (No. 1). The true form of the section of any cone cut thus, will be an ellipse (No. 3). This may be found by the intersections of the parallels, from the several points 1 2 3 4 5, in Nos. 1 and 2; as shown; or by Fig. 54.

FIG. 165.

To draw the plan and elevation of a cone cut by a plane, parallel to the axis.

Let *a b c* (No. 1), be any given cone, and *d e* the plane or part cut off. Draw any convenient number of lines *d* 2 4 6 *e* (No. 1), parallel to *a b*.

No. 2 is a plan. Repeat this plan at No. 3, but turned into the position shown. Find the points 6 4 2, etc., in *g h* (No. 3), by means of the circles drawn from *f* as a centre, with the widths from 1 3 and 5 to the centre line *c f* (No. 1), as radius. Draw the outline of the cone in No. 4, and from the several points in *d e* (No. 1), and *g h* in No. 3; draw lines cutting each other in *d* 2 4 6, etc. (No. 4.) Join these points by a curved line, and shade the part enclosed. The true form of this section is a hyperbola.

FIG. 166.

To draw the plan and elevation of a cone cut by a plane, parallel to one side.

Let *a b c* (No. 1), be the entire cone, and *e f* the cutting plane parallel to *b c*. Proceed as before, by drawing horizontal sections, 2 4 6 (No. 1), and their plans *e* 1 3 5 (No. 2). Project the points in No. 1 to No. 2, cutting the circles in 8 6 4 2 *e*, etc. Join these points by a curved line, and shade the enclosed part, as shown in No. 2. To obtain the true form of the section of any cone, cut thus:—Use *e f* (No. 1) as a ground line and erect perpendiculars at the points *e* 2 4 6 *f*. Find the parallel widths of the points in the plan No. 2 on line 7 8, and transfer these to line 7 8 (No. 3). Draw perpendiculars at these points to cut the former in *e*, 2 4 6 8, etc. Then the curved line drawn through these intersections will be a parabola. The elevation (No. 5), is found in the same way as in the previous figure; that is, by the intersection of lines from No. 1 and No. 4.

NOTE.—No. 4 is only a repetition of such parts of the plan (No. 2), as are required for the view (No. 5).

FIG. 167.

To draw the plan and elevations of any pyramid, cut at a given angle; also to find the true form of the section.

Let *a*, 1, 2, *b*, *c*, (No. 1), and *a* 1, 2, *b* 5, 6, *c* (No. 2), be the ordinary plan and elevation of a given hexagonal pyramid; and *e*, *f*, (No. 1,) the cutting plane. Mark the points 3, 4, etc., where the plane cuts the edges of the pyramid; and project all these points into the plan, to cut the lines *a c*, 1 *c*, 2 *c*, etc. Join those several points *e*, 4, 3, *f*, etc., and shade the enclosed space. At *e* 4, 3, and *f* (No. 1) erect perpendiculars to the cutting plane. Draw the centre line *e f* (No. 3) parallel to *e f* (No. 1). Measure off the widths in No. 2 at 4, 4, and 3, 3; transferring them to No. 3. and joining the points where they cut the former parallels, as in previous figures.

Repeat the plan No. 2 at No. 4 (but turned round one quarter), as shown; or upon *a b* (No. 5), copy the width in No. 2, parallel to A B. Then perpendiculars to A B, at all these points, to cut horizontals drawn from the several points in No. 1, will give the outline of No. 5. Complete this second elevation by joining all the points.

FIG. 168.

To draw the plan and elevation of any prism, cut by a given plane; also to find the true form of the section.

Let *a b c d* 1 2 3 4 (No. 1 and No. 2) be a given hexagonal prism, and *g h*, the plane. From the points in *g h* (No. 1), draw lines to cut the plan. Shade the part in section. At *g* 5 6 *b* (No. 1) square off lines as before, and lay down the parallels of width at right angles to the last drawn lines in No. 3; and cutting them in the required points which, when joined, determine the true form of this section.

Repeat the view (No. 2) at No. 4, turning it into the position shown. Then lines drawn from No. 1 and No. 4 will cut each other in No. 5; giving a second elevation or front view of the solid so cut.

FIG. 169.

To draw the plan and elevations of a cube in any given position when it is cut by a plane.

Let No. 1 be the view of the base, and No. 2 the given position of the cube, with its cutting plane *g h*. No 1 and No. 2 must agree in position—so that No. 1 will give the width, No. 2 the heights and lengths of the solid (the measurements required for Nos. 3 and 4). Draw lines from all the points in No. 1 and No. 2, to cut each other in No. 3 and No. 4.

Mark the several points in No. 1 and No. 2, with corresponding letters. Find their respective points of intersection in Nos. 3 and 4. Join these points and shade the sectional parts.

To place any solid in a given position see Figs. 132 to 136.

ENVELOPES OF SOLIDS.

FIG. 170.

To draw the envelope of a tetrahedron, one face being given.

No. 1 is a view of the solid. Let *a b d*, (No. 2,) be the given face. On each of the three sides *a b*, *b d*, *d a*, draw a similar triangle; then *c e f* is the envelope.

If these four triangles 1, 2, 3, 4, be cut out of cardboard (the lines *a b*, *b d*, and *d a*, being cut partly through), and the whole folded together, it will form a model of the solid.

FIG. 171.

To draw the envelope of a cube or hexahedron, one face being given.

No. 1 is a view of the solid. Let *a b c d*, (No. 2,) be the given face. Draw six of these squares together, in the form shown; and the space enclosed by the outline is termed the envelope. No. 2, cut out of cardboard, and folded together, will form a model of the solid.

FIG. 172.

To draw the envelope of an octahedron, one face being given.

No. 1, is the view of the solid, and *a b c*, the given face. Draw eight equilateral triangles, each equal to the given face, *a b c*, and placed together in the form shown in No. 2, so that when cut out, and folded together, it forms a model of the solid. (See Fig. 147.)

FIG. 163

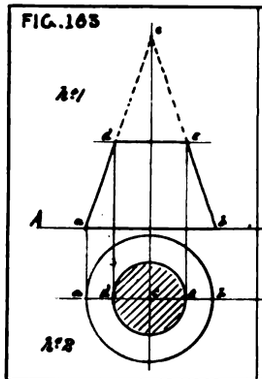


FIG. 164

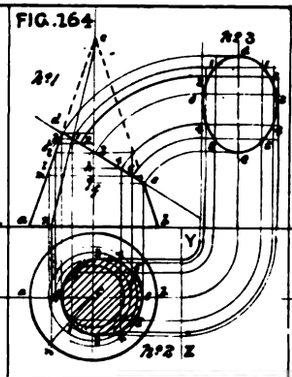


FIG. 165

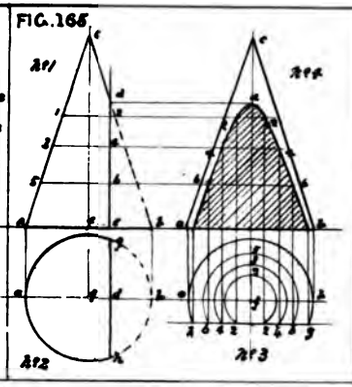


FIG. 166

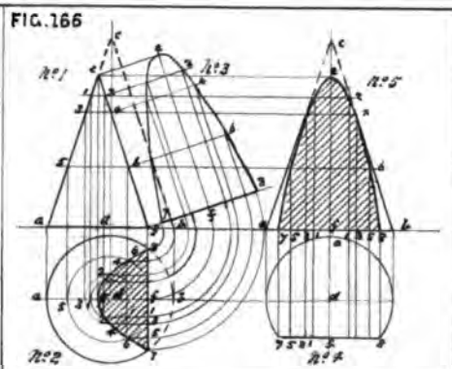


FIG. 167

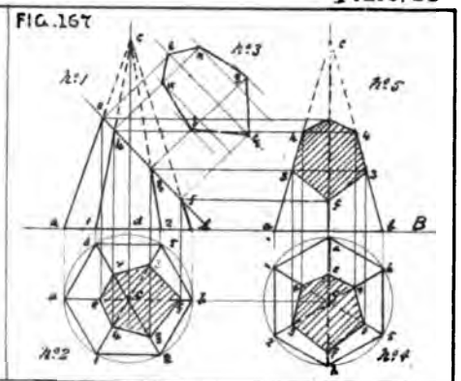


FIG. 168

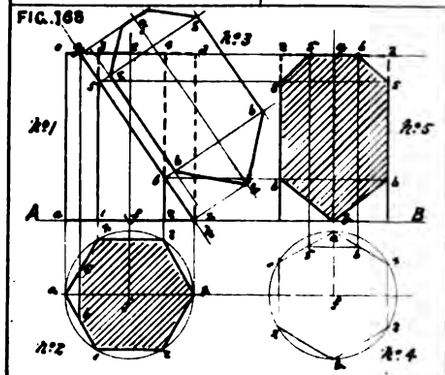


FIG. 169

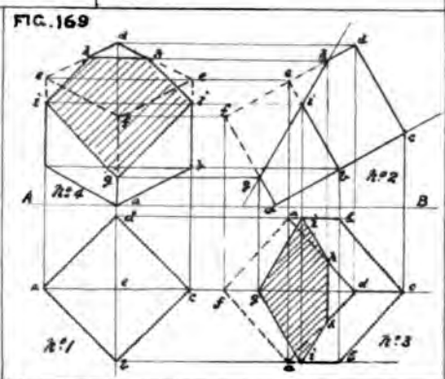


FIG. 170

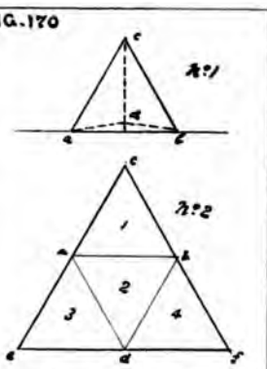


FIG. 171

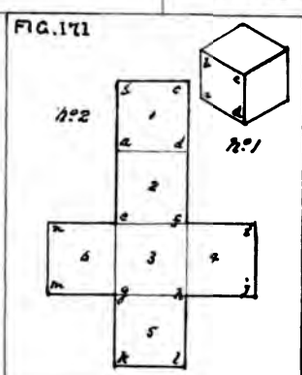


FIG. 172

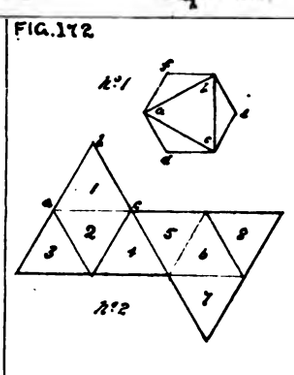


FIG. 173

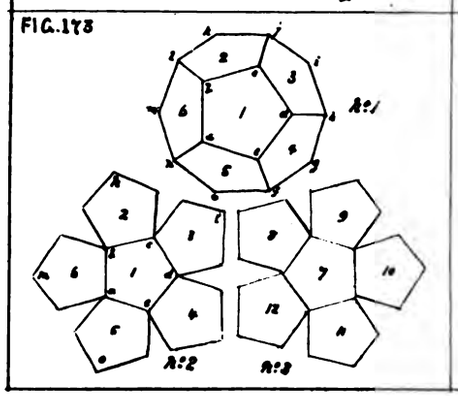


FIG. 174

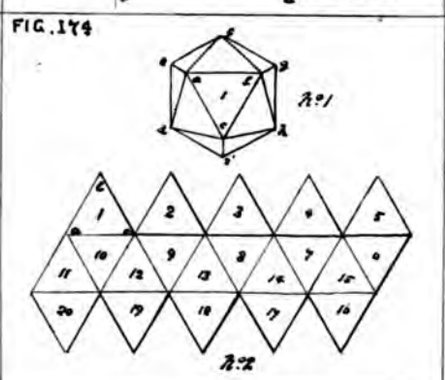


FIG. 175

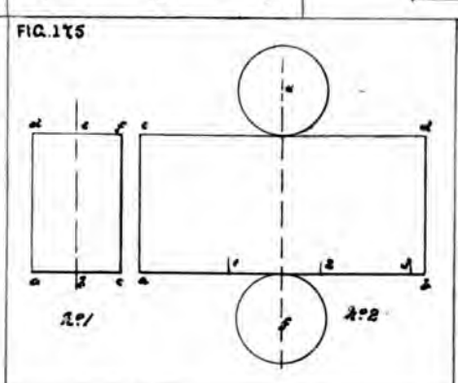


FIG. 176

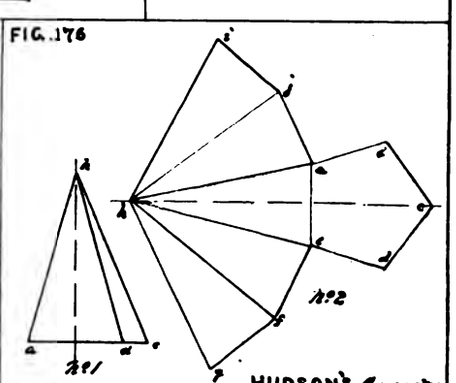


FIG. 173.

To draw the envelope of a dodecahedron, one face being given.

No. 1 is a view of the solid, and *a b c d e* (No. 2) the given face, which is a regular pentagon. (See Fig. 38.)

Upon each side of the given face, draw similar pentagons, in the manner shown (No. 2). Make No. 3 an exact copy of No. 2. Then these two half envelopes, cut out, folded and placed together, will form a model of the solid. (See Fig. 148.)

FIG. 174.

To draw the envelope of an icosahedron, one face being given.

No. 1 is a view of the solid, and *a b c* the given face, which is an equilateral triangle. (See Fig. 10, Plate 2). Draw twenty of these faces, in the form shown (No. 2).

FIG. 175.

To draw the envelope of any given cylinder.

Let No. 1 represent the given cylinder, *a c* being the diameter, and *a d* the length. By Fig. 45, find a line (*a b*, No. 2), equal to the circumference of the cylinder. Draw *a c*, equal to the length of the cylinder (*a d* No. 1), and at right angles to *a b*; also draw *c d* and *b d*, parallel to the former. Then this rectangle, with the addition of the two circles, *e* and *f*, will form the envelope of the solid.

FIG. 176.

To draw the envelope of any given pyramid.

Let No. 1 be the given right-pyramid, whose base (*a b c d e*, No. 2) is a regular pentagon. (See Fig. 38.)

At *a* and *e*, (No. 2,) as centres, with the length of one edge (*c h*, No. 1), draw arcs to cut at *h*. Draw lines *a h*, and *e h*, which represent one face. From *h*, as a centre, with *h a*, as radius, draw a circle, and upon it step off the other four sides, *f g i j*. Join these points with *h*, and with one another.

FIG. 177.

To draw the envelope of a given frustum of a cylinder.

The term frustum is applied only to solids, and means a piece, or fragment.

Let *a b d e*, (No. 1,) be the given frustum of the cylinder, *a b c d*. Upon the base, *a b*, draw a semicircle, and divide it into a convenient number of parts, 1 2 3. Draw up lines from these points perpendicular to the base, and cutting the plane of section (*e d*), in *f g h*. Find a line *a b*, (No. 2,) (by Fig. 45), equal to the circumference of the cylinder, and place it on a level with *a b*. (No. 1.) Divide *a b*, (No. 2,) similarly to the semicircle (*a*, 1, 2, 3, *b*, No. 1). Mark the several points with figures, and erect perpendiculars at each. From *e f g h* and *d*, No. 1, draw lines parallel to *a b*, cutting the former lines in *e f g* etc. (No. 2.) Draw a curve line, joining all these, and thicken the outlines. At any point on *a b*, draw a circle, *p* for the base; and at No. 2, set up *i r*, equal to *e d* (No. 1). Upon this line, mark also the positions of *f g* and *h* (No. 1). Mark off on each side of the centre line, distances *g s*, and *t n*, equal to 5 2, and 6 3, (No. 1). Draw the lines, as shown, to cut in points, which, when joined by a curved line, form an ellipse. The ellipse may also be found by Fig. 54.

FIG. 178.

To draw the envelope of a given sphere, to be composed of any number of equal and similar parts.

Let *a b* (No. 1) be the diameter of a given sphere to be divided into ten equal parts. Find a line *e* 10 (No. 2), equal to the circumference of the circle. On each side of it, at a distance equal to one quarter of the circumference, draw *a b* and *c d*, parallel to it. Divide the centre line into ten equal parts. Through the centre of each of these divisions draw lines to cut *a b* and *c d*. By Fig. 22, Plate 2, find the radius of an arc, to coincide with the three points *a e c*. Repeat these arcs at the several points *e f g h*, etc. The view (No. 1), is completed as follows:—Divide the semicircle, *a b*, into half the number of equal parts that the envelope contains; and, from the points 1 2 3, etc., draw perpendiculars to, and cutting *a b* in points, giving the position of the lines on the sphere.

FIG. 179.

To draw the envelope of a given frustum of any prism.

Let No. 1 be the given frustum, and *a b c d e*, its base. Draw a line *a b* (No. 2), and upon it set off the sides of the base, *a b*, *b c*, *c d*, and *e a*. At each point erect a perpendicular. From *m l* and *j* (No. 1), draw lines parallel to the base, and cutting the former at 14, 13, 8, 9, 15 and 16. Join all the points with thick lines. Subdivide the middle space, and draw a centre line. Upon this line set up distances to 10 and 11, equal to *j l* and *j m*. Repeat the base 2, 3, 5, 6, 7 *a b*, and draw parallels from 5 and 7, to cut the horizontal 10 12. Join the points 9, 10, 11, 12, 8.

FIG. 180.

To draw the envelope of a frustum of any cone.

Let *b c f e* (No. 1), be the given frustum of the cone, *a b c*. From *a*, as a centre, with *a b* and *a e* as radii, draw two circles. Find the circumference of the cone's base *b c*, and mark it upon the outer circle, from *j* to *l*. Draw lines from *j* and *l*, to the centre *a*. Add two circles *m* and *o*, equal to *b c*, and *e f*. Then these two circles, together with the annular part *j h l k g i*, will form the envelope or model of the solid. If the cone is cut obliquely, *o* will be an ellipse, and the inner circle *i g k*, a curved line, as in Fig. 182.

FIG. 181.

To draw the envelope of a given hemisphere, formed of any number of annular parts.

Let *a b c* be a section of the given half-sphere, to be formed of four parts. Divide the semicircle *a c b*, into eight equal parts, and figure them 1 2 3, etc. Draw a centre line *d c*, produced. Join *a* 1, and produce it, *a h*, to cut the centre line. Draw 1, 2 produced, to *f*, and 2, 3 to *e*. Then from *c e f*, etc., as centres, draw the circles *a* 1 2 3, and upon them mark their respective circumferences. Join their extremities with their several centres. These four parts, A, B, C and D, together, form the required envelope. This rule is useful to builders, and others, in covering domes.

FIG. 182.

To determine lines and points on the envelope of a cone; also to transfer them on to the cone.

Let *a b c* (No. 1), be a given cone, having upon it the two lines *a e* and *l r*, and the point *k*. Draw *b* 6

(No. 2), the envelope of one-half the cone; or, if necessary, draw the whole envelope. Upon the base, *a b* (No. 1), draw the half circle, *a* 3 *b*, and mark any convenient number of parts in it 1, 2, 3, 4, 5. From these points square up lines to the base, and from thence draw them to the apex *c*. At the several points where these lines cut the given ones, draw lines *f g h i j* and *m n o p q*, parallel to *a b*. Find the position of all these lines on the envelope, and through the points where the lines 1 2 3 4 5 6 (No. 2), cut the arcs drawn from *c*, as a centre, trace the lines *m g* and *g a*, which will be the form of the given lines on the envelope. The point *k* on the cone is found in the envelope in the same manner. If the lines and points are given on the envelope, reverse the process and find their appearance on the cone.

SHADOWS AND CAST SHADOWS.

NOTE.—In this division, as in Mechanical and Architectural Drawing, the light is always supposed to strike the object in parallel rays, the light being assumed at an infinite distance. Should it be required to draw the shadows as cast from a given point, the same principles will apply, with the exception that the lines must all converge to the given point of light. The direction of the light is generally shown as falling on the object at an angle of 45° both in plan and elevation.

FIG. 183.

To find the shadow on, and cast shadow of any sphere, the direction of the light being given.

Let *b l f c* (No. 1) be the sphere, and the angle of the light 45° in plan and elevation. Draw a tangent A B, at 45°, for a ground line, and a perpendicular to it through the centre *a*. Upon this line, at *a* (No. 2), as a centre, draw the plan of the sphere. At *a* (No. 3), in a line with and above the plan, repeat the sphere for the elevation. At No. 1 draw the tangent *c d*, at an angle of 45° to the ground line, A B (the angle of the light). Draw lines parallel to *c d*, through *a* and *l*, cutting A B in *j* and *k*. From *d j* and *k* draw lines perpendicular to A B. Draw parallels to A B, from No. 2, to cut at *j j* and *d*, giving the limits of the shadow, which is an ellipse. Then *m d* and *j j*, being the two diameters of an ellipse, the curve may be found by Fig. 54, or by points as follows:—Join *c l* (No. 1). Draw any line, as *e f*, parallel to A B, and imagine this to be a horizontal section, cutting *c l* in *g*. Draw *g i* parallel to *c a*, and *i i i* and *g g g* parallel to *a a*. Describe a circle from *a* (No. 2), as centre, to represent the plan of the section *e f*. At the points *g* and *g*, where *g g* cuts the circle, draw lines, *g i*, parallel to *a d*, and cutting *i i* in *i* and *i*, these points marked off equi-distant on each side of the centre line *j j*, gives four points in the ellipse. Other points may also be found in the same way by a number of these sections. Referring again to No. 1, the rays of light strike the sphere at *c* and *l*, leaving one-half in shade. Then by Fig. 140 find the projection of the circle *c l* (No. 1), in No. 2 and No. 3, and shade the parts, as shown.

NOTE.—The elevation (No. 1) is only used for determining the lines, but must always be drawn on a plane parallel to the direction of the light; that is, A B (No. 1), and *a d* (No. 2), should be parallel to one another, and to the path of the light in the plan.

FIG. 184.

To find the line of shadow in any grooved pulley.

Let the outline of two pulleys (No. 1, No. 2, and No. 3) be given, each having the same diameter. No. 1 is a section at *e f* (No. 2), but also serves as a plan for No. 3. Draw any horizontal line *h i* (No. 2), with half this as radius from Y (No. 1) as a centre. Describe a circle from *y* (No. 2); draw a line parallel to the direction of the light (45°) cutting *h i* in *x*. Draw X Z (No. 1) at the same angle as the light, and from the point *x*, opposite to *x* (No. 2) as a centre, with *a* Y as radius, draw an arc to cut the former circle in *j*; then this point projected to *j* No. 2 gives one point in the required curve.

To save labour in drawing more parallel lines to *h, i* (No. 2) it is always important to find the highest and lowest points in the curve; as, in most cases, the remainder of the line of shade may be traced in from such data. To find the highest point, *e* No. 2, draw a line from *a*, at an angle of 35° 16′, to *a b*, and at the point *e*, where it cuts, strike the horizontal line *e, f*, and its circles in No. 1 (*e f*). Project the point *g*, when the centre ray of light Z X (No. 1) cuts the circle to *l* (No. 2), the point required. The points in the curve in No. 3 are found in the same manner as No. 2. This principle is also explained in Fig. 187, and equally applies to this figure.

FIG. 185.

To find the line of shadow upon a cylinder cast from a square capping.

Let the outline of No. 1 and No. 2 be the plan and elevation of the given compound solid, the direction of light being assumed at 45°. Draw centre lines A, A, B, *e f*, No. 1, also *a d*, at 45°, and *g h* perpendicular to *a d*. From *a*, and *o* No. 1 draw up lines into No. 2. Draw a line from *a* (No. 2) at 45° to cut the other in *o*, the shadow of the corner *a*. Draw *g l* and *k j* No. 1 parallel to *a* A, and project these points into No. 2. At the points *j l* (in *a b*, No. 2), draw parallels to *a o*, cutting the former lines in *k* and *g*, which will give two points in the curve. Any other point, as *n*, may be found in the same way. Join all the points found with a curved line, as shown *o n k g*. The part *g f h* (No. 1), opposite to the light, will be in shadow, and is shown in No. 2 by line *g g*.

FIG. 186.

To find the line of shadow upon a cylinder cast from a hexagonal capping.

Let the outline of No. 1 and No. 2 be the plan and elevation of the given compound solid, the direction of the light being assumed at 45°. The method described in the previous figure applies also to this—that is, the lines in No. 1 representing rays of light, being projected into No. 2, and re-cut by parallels at 45°, give the points *s r o k*, etc., in the curve required as indicated.

FIG. 177

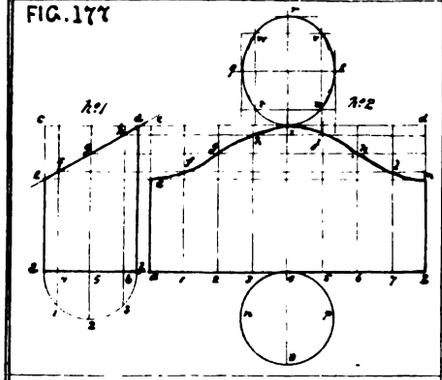


FIG. 178

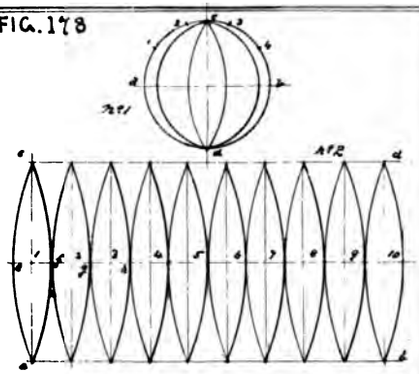


FIG. 179

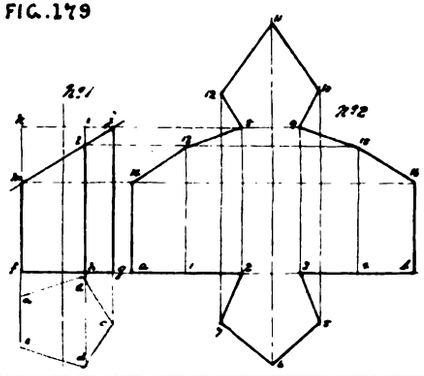


FIG. 180

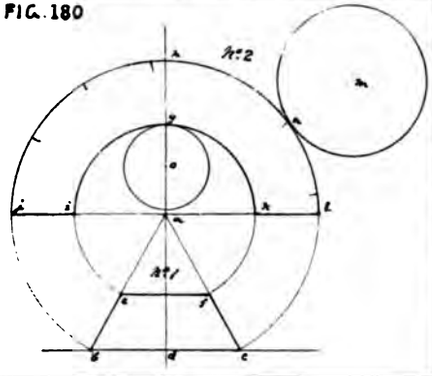


FIG. 181

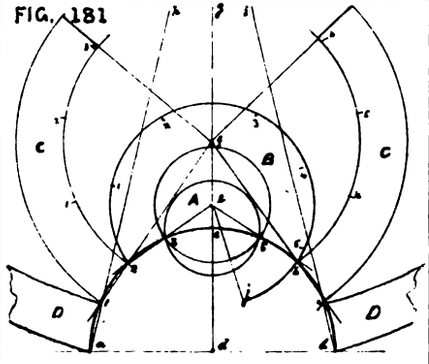


FIG. 182

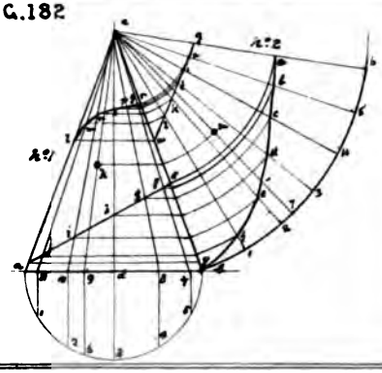


FIG. 183

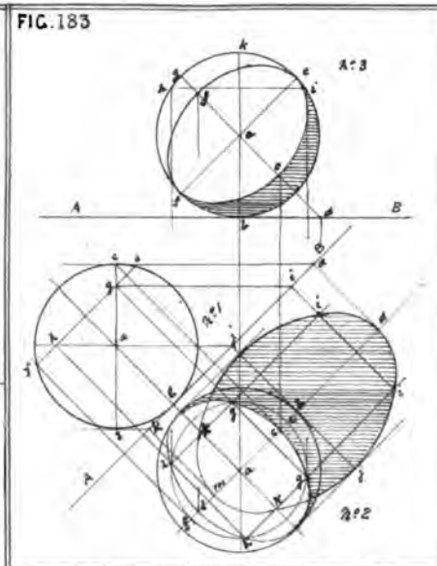


FIG. 184

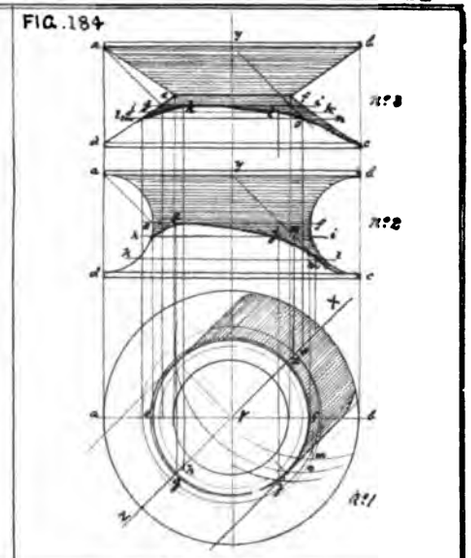


FIG. 185

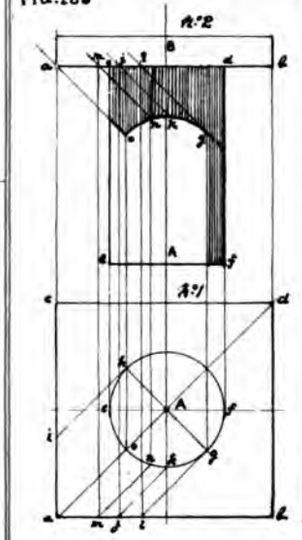


FIG. 186

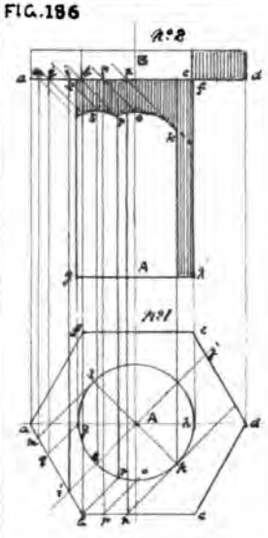
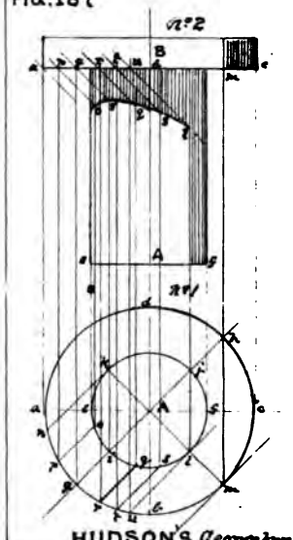


FIG. 187



HUDSON'S Geometry.

FIG. 187.

To find the line of shadow upon a cylinder thrown from a round capping or collar.

Let the outline of No. 1 and No. 2 be the plan and elevation of the given solid, the angle and direction of the light being as before. In No. 1, draw any lines, *o p*, *i g*, *q r*, *s t*, and *t u*, all parallel to *g h* (the direction of the light). From each of these points in No. 1 draw up lines into No. 2, and from *p g r t u* (No. 2) draw lines at 45° to *a c*, cutting the lines belonging to the inner circle, in *o i q s l*, through which draw a curved line representing the form of the shadow. A line *k m* (No. 1) at right angles to *g h*, and passing through A, gives points *l* and *m* the last rays of light; the parts to the right of these points are in shadow, as shown in No. 2. The highest point *i* (No. 2) in the curve is projected from *i* (No. 1).

FIG. 188[1].

To find the shadow cast from a slip, A B C D, upon a parallel and vertical plane, Y Z.

Let Y Z (No. 1) be the plane, and A B the slip, A B C D (No. 2) being directly over its plan (A B No. 1), and W X Y Z is the vertical plane. Draw lines from A and B (No. 1) at 45°, to cut Y Z in *a* and *b*. From these points project lines into No. 2. Draw lines from A B C and D (No. 2) at 45°, to cut the others in *a b c* and *d*. Join these points and shade the enclosed space, which is the form of the shadow.

Whenever the object is parallel to the plane, its shadow cast thereon is equal and similar.

Therefore, in drawing the shadow of any irregular object, much time may be saved by finding one or two points and drawing the lines parallel to those of the object.

FIG. 189.

To find the cast shadow when the object is not parallel to the vertical plane.

No. 1 being the plan, and No. 2 the elevation, A B and A B C D the object as before, and Y Z the plane, proceed as in the previous figure.

FIG. 190.

To find the shadow cast upon an irregular surface by an object parallel to that plane.

Let Y Z (No. 1) be the plan of a given plane, having upon it a number of mouldings. A B and A B C D is the object in plan and elevation, W X Y Z (No. 2) being the plane, with its mouldings in elevation. Draw any lines through A B (No. 1), at 45°, to cut the plane; for instance, those at *i j k l m*, etc., and coinciding with the corners of the mouldings *d e f g h*. From all these points in A B and Y Z, draw up lines into No. 2. Then lines from the intersections in A B (No. 2), at 45°, will cut those from Y Z in points which, when joined, give the outline of the shadow required.

FIG. 191.

To find the shadow cast upon any irregular surface when the object is at right angles to that plane.

Let the slip A B (No. 1 and No. 2) be any given object, and Y Z the plane, in plan and elevation. Proceed as in previous figures, and it will be seen that, when the object and plane are under these conditions, that is, parallel and perpendicular to the ground line, the form of the shadow is not affected by the interception of the mouldings. It will only be necessary to find the length of the shadow, and draw the lines from A and B (No. 2) at 45°.

FIG. 192.

To find a shadow cast upon any double inclined surface.

Let the slip A B (No. 1) and A B C D (No. 2) be the plan and elevation of any given object, and Y *c* Z (No. 1) the planes. Draw any lines from A and B (No. 1), at 45°, to cut the planes in *a* and *b*, and another from *c*, to cut A B in *d*. From all these points draw up lines into No. 2. Then lines drawn from A B C D *g* and *h* (No. 2), at 45°, will cut the former in *a b c d e* and *f*. Join these points, giving the form of the shadow.

FIG. 193.

To find a shadow cast upon any curved surface.

Let the slip A B (No. 1), and A B C D (No. 2), represent any given object, and X Y Z the curved plane. No. 1 is the plan and No. 2 the elevation. Lines drawn at an angle of 45° from any number of points in A B (No. 1), and the same points in A B C D (No. 2), intersect lines drawn up from *a c d e b* (No. 1), giving the form of the shadow required, as in previous figures.

[1] In the following Figures on Shadows, a simple slip is taken for an example; but the Shadows of any Plane Figures or Solids are found in the same way.

ISOMETRICAL PROJECTION.

ISOMETRICAL Projection is a kind of distorted perspective, or bird's-eye view[2]. It is much used by architects, builders, masons, engineers, etc., on account of its giving a more comprehensive idea of the object than any other form of drawing. It has the advantage of showing three views in one, and is easily drawn and measured to scale.

All the lines that are parallel in the object will be parallel in the view.

Three faces of any solid are shown equally inclined to the plane of projection (or paper); therefore, in representing a cube, three of its faces will be equal, and its edges form angles of 60° and 120°. In a cube, the edges are drawn vertical and inclined at 30° to the right and left. Most of the lines can be drawn by the aid of a "straightedge" and "setsquare," having angles of 30° and 60°. The view may be commenced at any point, but it is advisable to select the vertical centre line, marking off the heights and centre lines of the faces.

This kind of Perspective Drawing should never be applied to architecture, except in details or portions of buildings; for instance, in representing a street, or any extended object, the distortion is at once apparent.

FIG. 194.

To represent any solid, as an obelisk, in Isometrical Projection.

Commencing at any point, as *a*, draw *a b* and *a c*,

[2] Invented by Professor Farish, of Cambridge.

inclined in opposite directions at 30°, and mark off their true lengths along these lines. Draw a vertical line at *a*, and mark off its length *a e*. From *e*, draw lines *e d* and *e f*, parallel to *a b* and *a c*, join *d f*. Find the middle point *z*, in *e f*, and from *z* draw a line parallel to *e d*, cutting *f* in *k*, the centre of one face of the bottom plinth. Erect a centre line *k t*, which in this instance coincides with the front corners. From *k*, mark off a length, *k y* equal to half the width of the second block. Through *y*, draw *g h*, parallel to *e f*, and cutting the former lines. Draw *g i*, parallel to *e d* and *y x*, vertical (the height of the block No. 2). Set off *x w* at 30°, and equal to the projecting part of the block No. 3. Draw a vertical *w v*, equal to the depth of the block (No. 3). Through *w*, draw *k l; o p* through *v*, and *k m, o n*, parallel to *a c*, marking off their correct lengths. Join *l p, k o, m n*, etc., with vertical lines. Find the width *v u*, and through *u* draw *q r*. Join *n p*, and complete the base of the pyramid No. 4. Find the height of the pyramid from the diagonal *n p* of the base, to the point *t*, and join *q r* and *s* with *t*. The diagonals *d f* and *n p* are not actually necessary, but assist in detecting errors.

FIG. 195.

To represent in Isometrical Projection three intersecting planes, each at right angles to the other two.

Commencing at any point, as A, draw A B C vertical, and A E, A D, inclined in opposite directions at 30°; these lines making angles of 120° with each other. Mark the lengths of the planes upon these lines. Through B C, D and E, draw lines, each parallel to one of the former. Join all the extremities of the lines, completing the figure as shown.

FIG. 196.

To represent in Isometrical Projection any cube having an inscribed circle on each face.

The cube may be drawn by means of a circle, dividing it into six equal parts for the corners A B C E F G. Commencing at D, the centre, draw D A and D C at 30°, and D E vertical. Draw A B, B C, C F, F E, F G, and G A, each of which will be parallel to one of the first drawn lines. The inscribed circles, which are ellipses, may be drawn by means of two diameters, H I and J K (No 1 face), by Fig. 54, or as follows:—Upon one side, as A G, describe a semi-circle A *c* G, and a rectangle A *a b* G. Draw lines from the corners *a* and *b* to the centre H. Through the points *d* and *e*, where these lines cut the semi-circle, draw parallels to either of the sides of the rectangle. From each corner of each face, mark off distances equal to *a g*, and draw lines parallel to the sides. Mark the points where these lines cut, as *k l m j*, through which draw a curved line for the ellipse. Repeat this in the faces No. 2 and No. 3. The planes of the three faces are each equally inclined to the plane of the projection.

Those lines which are parallel to any of the planes are either inclined at 30° to the right or left, or drawn vertical.

All circles become ellipses.

FIG. 197.

To represent in Isometrical Projection any rectangular object, as a piece of masonry or brick-work foundations of given dimensions.

Commencing at any point, as the centre *k*, draw lines *i j* and *l m* through *k* at 30°. Measure off on these lines the widths and lengths. Through these points draw lines parallel to the first, giving all necessary lines for the top face. Rub out the parts of the lines which may cross one another. Drop a vertical at each corner, and mark its required length. Mark off the footings in the same way, or as at *n o p j s t*, etc. To insure accuracy at any complicated part, as the buttresses, find the profile, as at *j s t*, with cross lines equal on each side. Complete the figure as shown, rubbing out all construction lines.

NOTE.—These four examples will be found sufficient to show the method of representing any object, either simple or intricate.

PARALLEL PERSPECTIVE.

PARALLEL Perspective is so called from the position to the object with regard to the plane of the observer, the principal planes of the object being either parallel or perpendicular to that of the observer. The *point of sight* is identical with the position of the observer, and is generally placed opposite to the centre of the object; or, if to one side, the operation is the same. The greatest angle of vision is said to be about 70° (that is, without turning the head), therefore the position of the point of sight must always be sufficiently far from the object to contain every part within that angle, otherwise the view will appear distorted. The *plane of the picture* is that plane in which all the rays cut, in their passage from the object to the eye of the observer.

The *plane of the picture* is invariably shown at the nearest point of the object; as, if it were placed nearer to the point of sight, it would only have the effect of reducing the view proportionately, which can be regulated better in the first instance by scale. A knowledge of scale-drawing is necessary; this may be acquired by a little practice, and reference to Fig. 52. The height of the *point of sight* is five feet above the ground line, to agree with the average height of a person.

The nearer the *point of sight* can be brought to the *plane of the picture*, the more will be shown of the interior.

FIG. 198.

To draw the perspective view of an interior of any building.

Let No. 1, C D E F, be the plan of a small schoolhouse, the point of sight being at A, and C D the plane of the picture. Then draw lines from every part of the plan towards A, and cutting C D. From all these points in C D draw up perpendiculars for the elevation.

Draw the outline elevation of the building, No. 2, marking the heights of doors, windows, etc., upon

FIG. 188

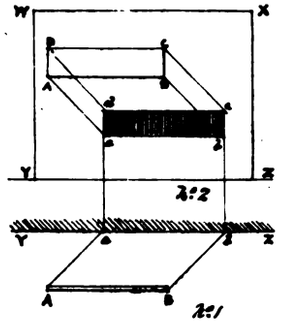


FIG. 189

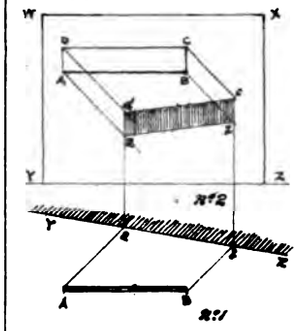


FIG. 190

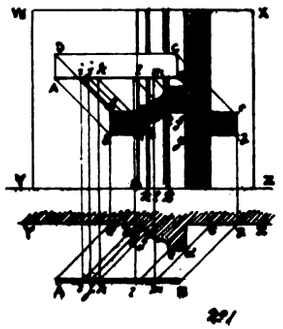


FIG. 191

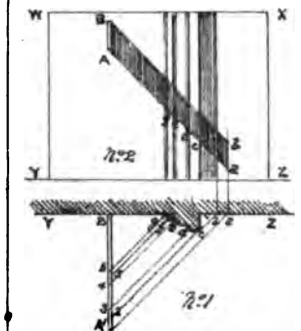


FIG. 192

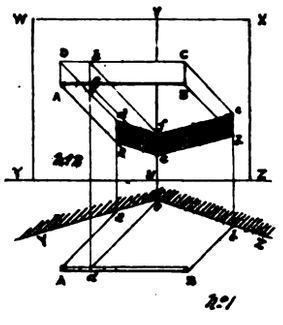


FIG. 193

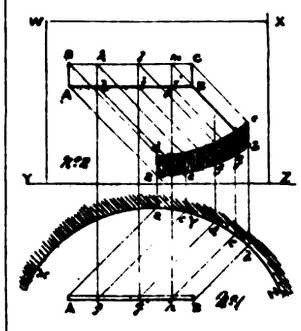


FIG. 198.

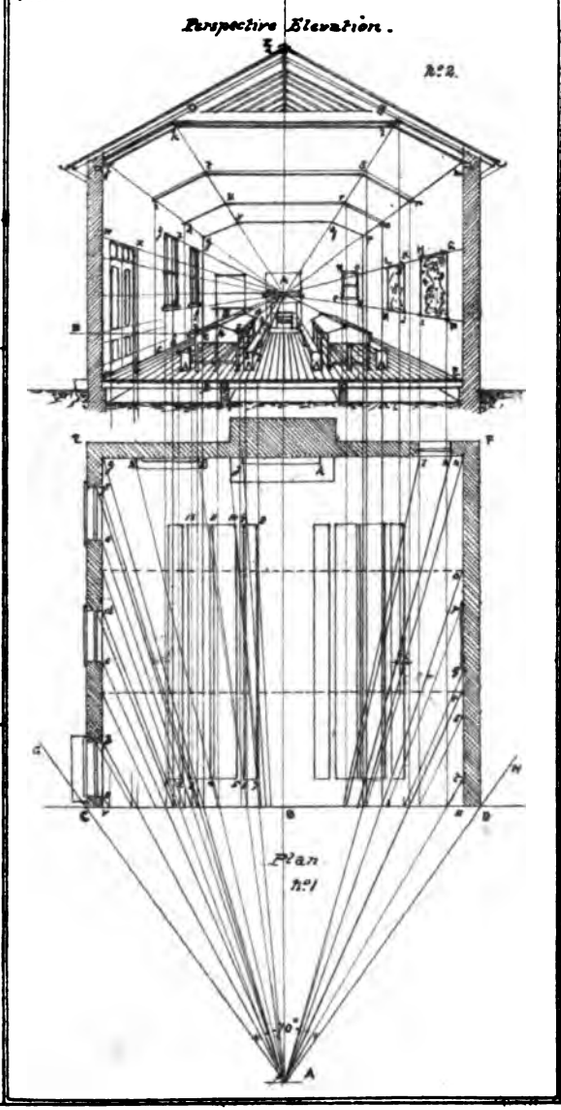


FIG. 194

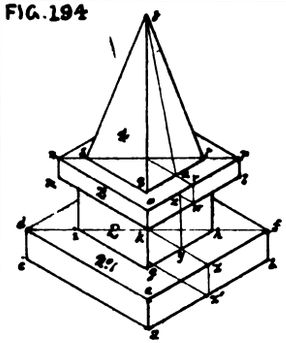


FIG. 195.

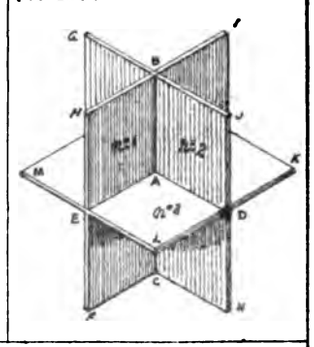


FIG. 196

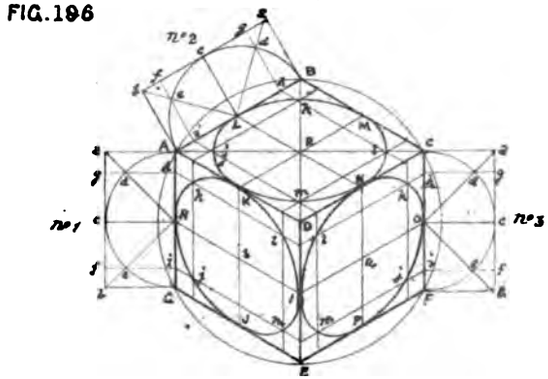


FIG. 197

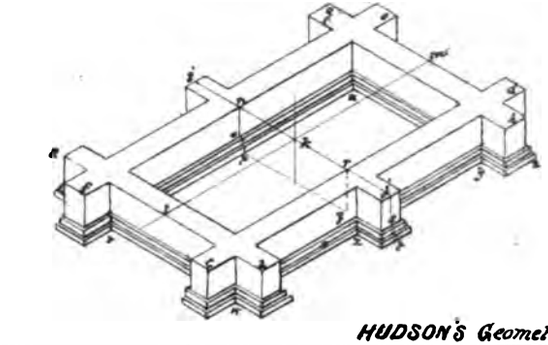


FIG. 199.

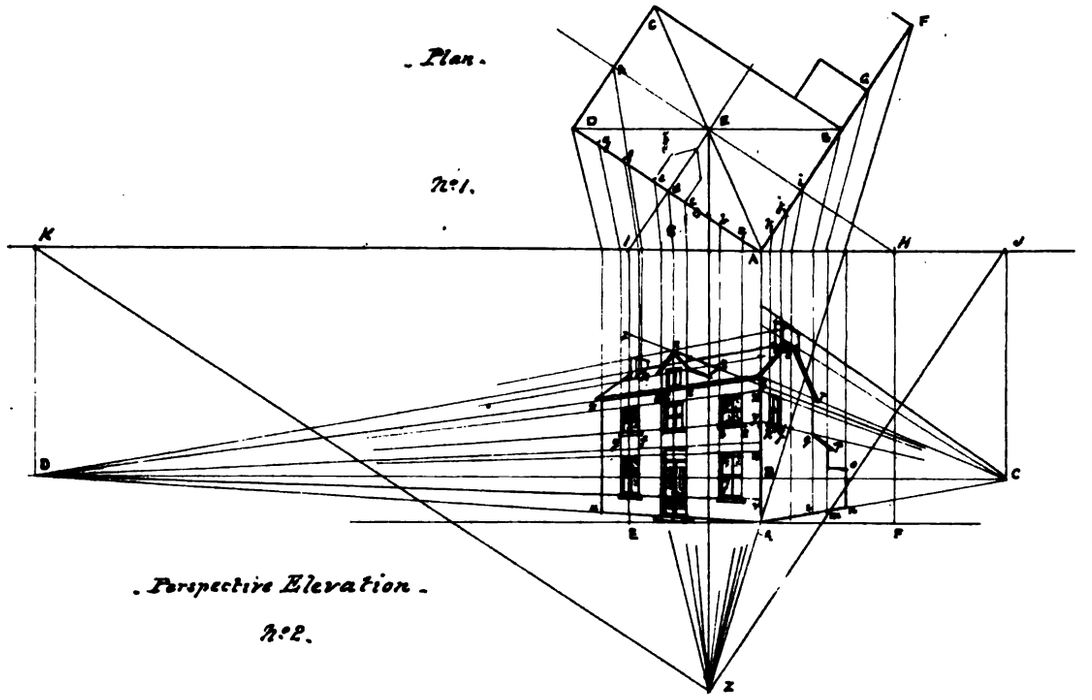
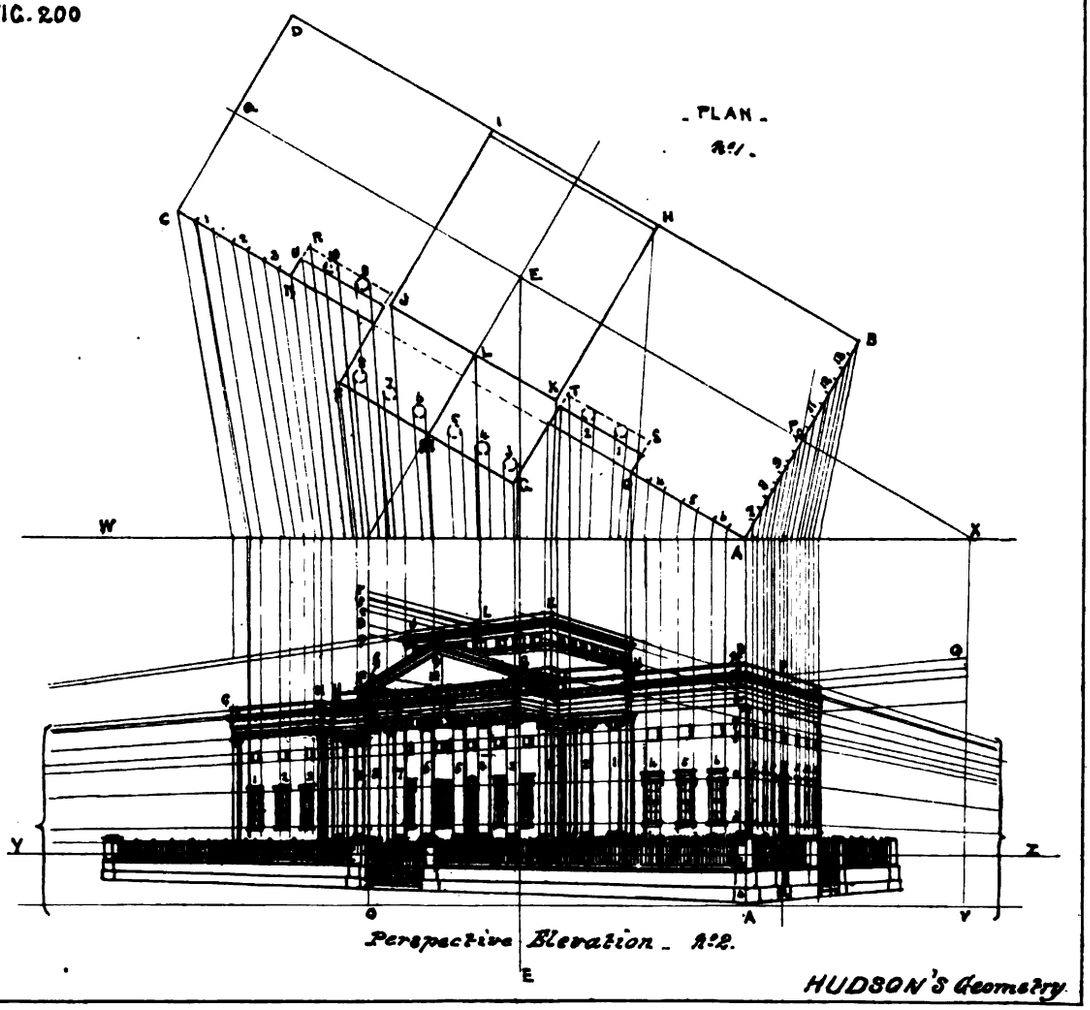


FIG. 200



these lines. Fix the position of the point of sight A (No. 2), vertically over A (No. 1), and at five feet above the ground line (*a* E). Draw lines from every point in the outline, or profile, to A, cutting the verticals from C D (No. 1), in points which, when joined by thick lines, give the main features of the view. All those lines originally vertical will coincide with those drawn up from C D (No. 1). The positions of the forms and desks are found in a similar manner; thus, the parts marked 1 2 0 4 5 6 7 (No. 1) are carried forward to C D; from thence, upwards, their heights then marked from the ground line, and joined with A (No. 2). Where the lines from 1 2 3, etc., to A (No. 1), cut C D, draw up lines to cut the former (drawn to A) in 1 2 3, etc. (No. 2.) The construction lines shown in the figure need not be drawn after the method is understood, as the points in C D (determined by the "straightedge") are all that is required in the plan; and many of the lines in the elevation may be omitted, the projected points only being necessary. Some of the details may be filled in without finding their true projection. In the point of sight, A, a needle should be fixed, against which the "straightedge" is placed.

OBLIQUE OR LINEAR PERSPECTIVE.

OBLIQUE Perspective differs from *Parallel Perspective* in having three *vanishing points* instead of one, *the plane of the picture* not being parallel to the object.

Most of the remarks upon *Parallel Perspective* apply equally in this case, except that in *Oblique Perspective* the point of sight is moved as far as possible from the object, so as to show more of the exterior.

FIG. 199.

To draw a perspective view of a detached dwelling, the position of the point of sight and the building being given.

Let Z be the point of sight, and A B C D, etc. (No. 1), the plan of the building to any convenient scale. Draw J K through A for the *plane of the picture.* From Z draw lines parallel to the sides of A D and A F, cutting at J and K. Draw a line from A for the elevation, No. 2, and upon it mark off all the heights of the building from the scale, E F being the ground line; set up 5 ft. of the scale from A to B (No. 2), and draw the horizontal, C D. Mark the points C and D under J and K for the vanishing points. From A *v w y z*, etc., on the line of heights (A A) draw lines to C and D. Draw lines from *a b c*, etc. (No. 1), towards Z, to cut the plane J K, and from thence draw verticals cutting the vanishing lines of No. 2. Thicken such of these lines which form part of the view, and join the points of the roof, etc. The dormer window may also be found by a separate line of heights, E *l* (No. 2), produced from E I (No. 1).

FIG. 200.

To find the perspective view of any large building.

The operation here is the same as in the last figure, but to embrace a larger view, the vanishing points Z Y to the right and left, also the point of sight, are not shown, being at a considerable distance.

In practice, the vanishing points seldom occur within the limits of the paper, and pins are placed in the board, against which the "straightedge" is kept, to draw the required portions of these vanishing lines belonging to the picture.

All planes that are parallel have the same *vanishing point.*

A B C D F G, etc. (No. 1), is the plan in outline of a given building. W X is the *plane of the picture.* The position of the observer or point of sight being somewhere on E E, produced, draw lines from every point in the plan towards this point, to cut W X. Draw the centre lines, E M and E P, to cut W X, from thence draw perpendiculars for lines of height in the elevation O *r* and Q V, also A B (No. 2). Upon these lines mark off from the ground line *o v* the heights of the various parts of the building. From the points in the line of heights, A A, draw lines to the right and left towards the *vanishing points* on the horizontal line Y Z. Then perpendiculars projected from the points in W X (No. 1) will cut the former and give most of the lines of the perspective elevation. The projecting entablature or portico is obtained from the separate line of heights, O *r* (No. 2), and the points K G M F projected from No. 1. The receding lines of the portico and other parts, N R G T and K H (No. 1), will all vanish to the same point Z, all being parallel planes. The railings are not shown in the plan, for want of space. After projecting the true outline and chief features of the building, a great deal remains to be filled in to complete the drawing, as the application of the method to all the details would become too complicated. A knowledge of free-hand drawing is essential, as much depends on the taste and ability of the student.

Other methods for determining the perspective elevation are to be found in books treating exclusively upon perspective, in which plans are not required; but such systems invariably fail to convey an adequate conception of the subject. When the perspective drawing is intricate, the plan and construction lines should be drawn on a separate piece of paper, the requisite points in the plane of the picture being transferred to the elevation, as these construction lines would only disfigure the drawing.

Perspective projection is the application of Geometrical principles to the Fine Arts.

In competitive and prize drawings for public buildings, the outlines in perspective are usually determined and drawn by the draughtsman, and finished by an artist.

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95. ,, an ellipse into an equal circle.
96. ,, a circle into an equal ellipse.
97. To construct any polygon equal to a triangle.
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175. ,, ,, cylinder.
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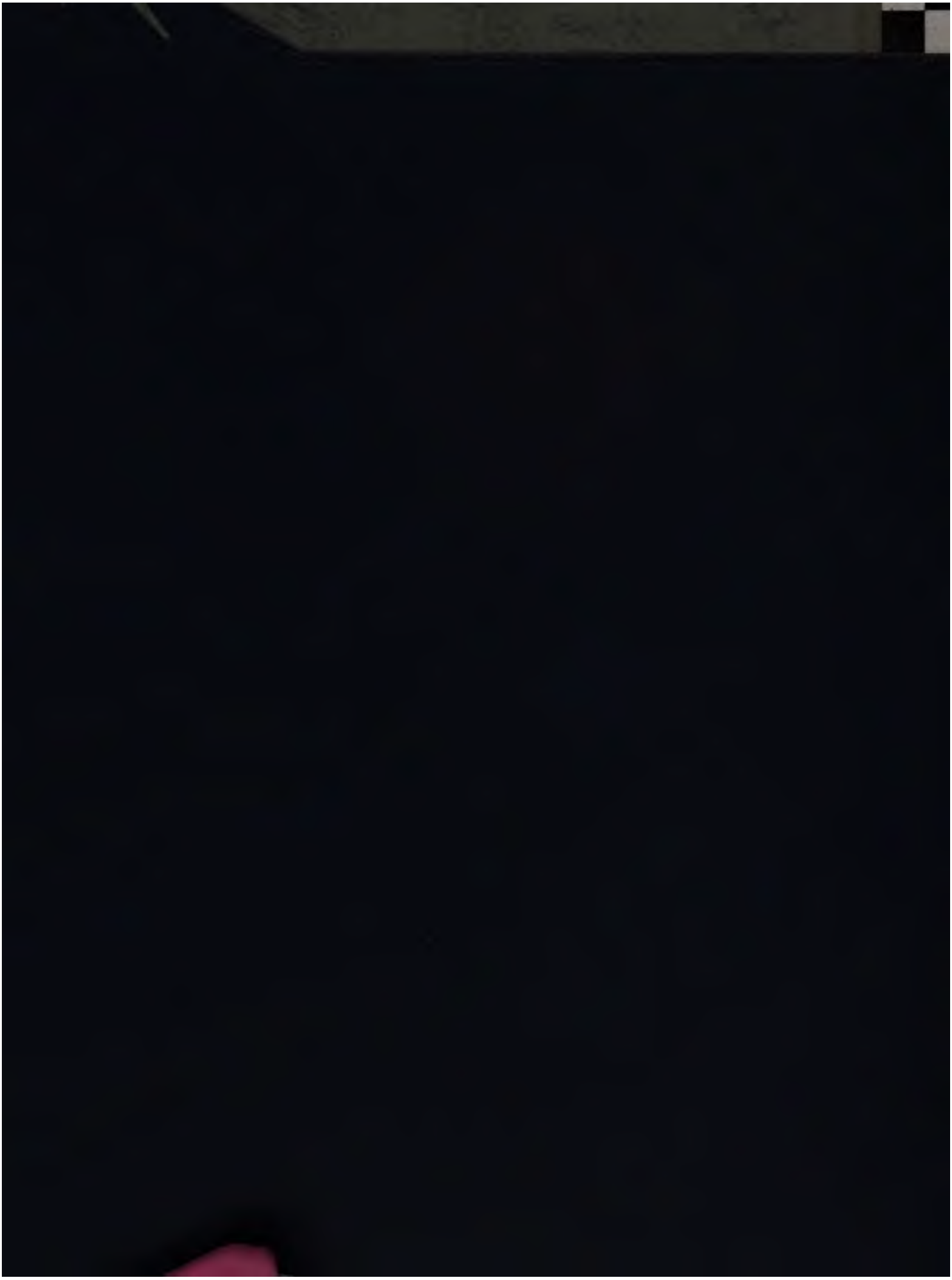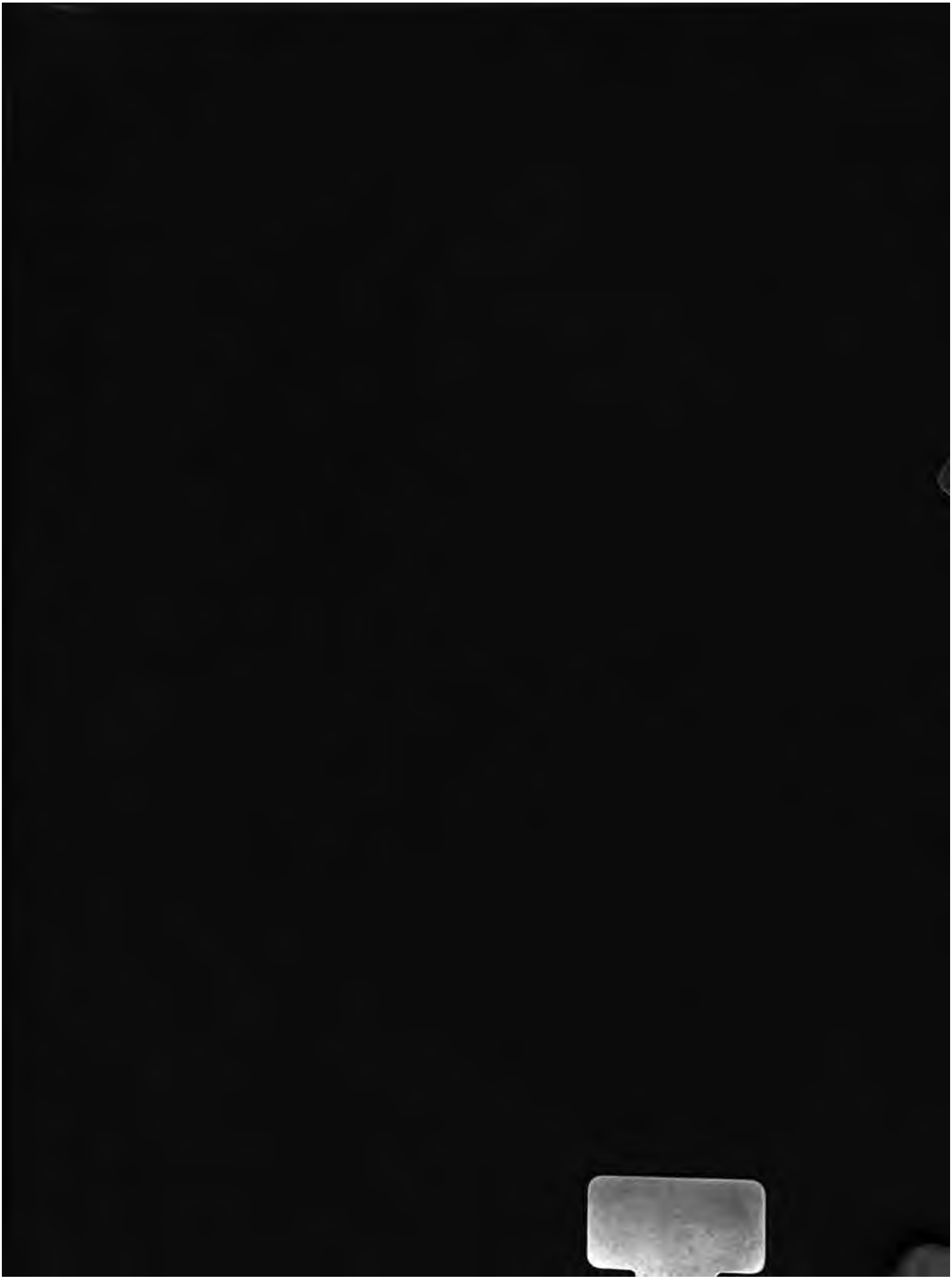
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